MATH 351 - 01 **Modern Algebra** **Homework Assignment 10**

Instructions

1. There are eight problems, worth 50 points. You must submit solutions by noon on May 3, 2022.
2. You may refer to lecture notes, the textbook, video lectures and lecture notes posted on Canvas.
3. You may not consult any external resources. This means no internet searches, materials from other classes or books or any notes you have taken in other classes. You may not use Google or any other search engines for any reason. You may not use any shared Google documents.
4. You may not discuss this homework assignment or questions related to the assignment with any person outside class.
5. Show ALL your work for full credit.
6. You must sign the front page of the homework assignment and submit it with your solutions. If you do not submit the signed first page, your solutions will NOT be graded and it will be worth ZERO points.
7. Please use A4 sheets to write your solutions and start each problem on a new sheet of paper. Please make sure your solutions are very clear and legible.
8. Please follow the guidelines given in the syllabus on how to prepare and how to submit your homework assignment.

All work submitted is mine and mine alone.

I have read and followed the instructions above.

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Signature

1. (2 points each) Find the orders of these permutations.
   (i) $(1,2)(2,3)(3,4)$
   (ii) $(1,2,5)(2,3,4)(5,6)$
   (iii) $(1,3)(2,6)(1,4,5)$
   (iv) $(1,2,3)(2,4,3,5)(1,3,2)$
2. (2 points each) Let the dihedral group D_n be given by elements a of order n and b of order 2, where $ba = a^{-1}b$.
   (i) Show that $a^{-m} = a^{n-m}$, for all $m \in \mathbb{Z}$.
   (ii) Show that $ba^m = a^{-m}b$, for all $m \in \mathbb{Z}$.
   (iii) Show that $ba^m b = a^{-m}$, for all $m \in \mathbb{Z}$.
3. (6 points) Find the order of each element of D_6.
4. (a) (6 points) List the elements of A_4.
   (b) (4 points) For any elements $\sigma, \tau \in S_n$, show that $\sigma\tau\sigma^{-1}\tau^{-1} \in A_n$.
5. (5 points) Let $\tau = (a,b,c)$ and let σ be any permutation. Show that

$$\sigma\tau\sigma^{-1} = (\sigma(a), \sigma(b), \sigma(c)).$$

6. (4 points) Show that if $n \geq 3$, then the center of S_n is trivial.
7. (5 points) For $\alpha, \beta \in S_n$, let $\alpha \sim \beta$ if there exists $\sigma \in S_n$ such that $\sigma\alpha\sigma^{-1} = \beta$. Show that $\sim$ is an equivalence relation on S_n.
8. (2 points each) Let permutations in S_4 act on polynomials in four variables by permuting the subscripts.
   (i) Which permutations in S_4 leave the polynomial $(x_1 - x_2)(x_3 - x_4)$ unchanged?
   (ii) Which permutations in S_4 leave the polynomial $\displaystyle\prod_{1\leq i<j\leq 4} (x_i + x_j)$ unchanged?
   (iii) Which permutations in S_4 leave the polynomial $\displaystyle\prod_{1\leq i<j\leq 4} (x_i - x_j)$ unchanged?