Please do not write in the boxes immediately below.

problem	1	2	3	4	5	6	total
points							

MATH 136 Fall 2023 Midterm Exam 3
November 30, 2023

Your name and section⎯⎯

The exam has 6 different printed sides of exam problems and 1 side workspace.

Duration of the Midterm Exam is 90 minutes. There are 6 problems, worth 10 points each. From Problems 1 – 6, only 5 problems will be graded. If you solve all Problems 1 – 6, you must cross out the problem in the box above that must not be graded. If you solve all Problems 1 – 6 and do not cross out a problem, only the first five problems will be graded. Show all your work for full credit. Books, notes etc. are prohibited. Calculators, cellphones, earphones, AirPods and cheat sheets are NOT permitted.

1.   (a) Show that

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{\pi}{2n} \sin\left(-\frac{\pi}{4} + \frac{i\pi}{2n}\right) = 0.$$

Hint: First show how to express the limit as a definite integral, then justify the value of the integral.

(b) **Set up** an integral to find the area of the region bounded by the graphs of $y = 8 - \sqrt{x}$, $y = \sqrt{x}$, and $x = 0$.

2.   (a) If $f(x) = \displaystyle\int_0^x (1 - t^2)\, e^{t^2}\, dt$, on what interval is f increasing?

   (b) A particle moves in a straight line with the given velocity (in m/s). **Set up** integrals to find the displacement and distance traveled over the time interval.

$$v(t) = 36 - 24t + 3t^2, \quad [0, 10]$$

3. (a) **Set up** an integral to find the volume of the solid obtained by rotating the region enclosed by the graphs about the given axis.

$$y = 2\sqrt{x}, \ y = x, \ \text{ about } x = -2$$

(b) **Set up** an integral to compute the arc length of $y = \ln(\sin x)$ for $\frac{\pi}{4} \leq x \leq \frac{\pi}{2}$. In your answer, the integrand must not involve a square root function. **Hint:** $1 + \cot^2 x = \csc^2 x$

4.   (a) Determine whether the integral is convergent or divergent.

$$\int_{-1}^{2} \frac{x}{(x+1)^2}\, dx$$

(b) Use the Comparison Test for Improper Integrals to determine whether the integral is convergent or divergent.

$$\int_{1}^{\infty} \frac{1+\sin^2 x}{\sqrt{x}}\, dx$$

5. Evaluate the integral.

$$\int \frac{10}{(x-1)^2(x^2+9)}\, dx$$

6.   (a) Show that $a_n = \dfrac{7n^2}{n^2 + 2}$ is increasing. Find an upper bound. Is the sequence convergent? Justify your answer.

(b) Prove that the following series diverge. Mention any theorem used.

(i) $\displaystyle\sum_{n=1}^{\infty} \dfrac{n}{\sqrt{n^2 + 1}}$

(ii) $\dfrac{0}{1} - \dfrac{1}{2} + \dfrac{2}{3} - \dfrac{3}{4} + \cdots$

WORKSPACE