Please do not write in the boxes immediately below.

| problem | 1 | 2 | 3 | 4 | 5 | 6 | total |
|---|---|---|---|---|---|---|---|
| points | | | | | | | |

# MATH 135 Fall 2023 Midterm Exam 3

November 30, 2023

Your name and section_____

*The exam has 6 different printed sides of exam problems and 1 side workspace.*

*Duration of the Midterm Exam is 90 minutes. There are 6 problems, worth 10 points each. From Problems 1 – 6, only 5 problems will be graded. If you solve all Problems 1 – 6, you must cross out the problem in the box above that must not be graded. If you solve all Problems 1 – 6 and do not cross out a problem, only the first five problems will be graded. Show all your work for full credit. Books, notes etc. are prohibited. Calculators, cellphones, earphones, AirPods and cheat sheets are NOT permitted.*

1. (a) Find $\lim_{x\to\infty} f(x)$ if for all $x > 0$, $\dfrac{4\sqrt{x^2+x+1}}{x+2} < f(x) < \dfrac{12e^x+1}{3e^x}$. Mention any theorem used.

(b) Determine where $f$ is continuous expressing your answer in interval notation. Show all your work.

$$f(x) = \begin{cases} 2^x & \text{if } x \leq 1 \\ 3-x & \text{if } 1 < x \leq 4 \\ \sqrt{x} & \text{if } x > 4 \end{cases}$$

2. (a) Use the limit definition to compute the derivative of the function $f(x) = \sqrt{x}$ at $x = 9$.

(b) Compute $\dfrac{d^{499}}{dx^{499}}(\sin x)$.

3. (a) Find equations of both lines that are tangent to the curve $y = x^3 - 3x^2 + 3x - 3$ and are parallel to the line $3x - y = 15$.

(b) For what values of $x$ does the graph of $f(x) = e^x - 3x$ have a horizontal tangent?

(c) Show that the curve

$$y = 2e^x + 3x + 5x^3$$

has no horizontal tangent.

4. (a) Show that the equation $x^3 + e^x = 0$ has exactly one real root. **Hint:** $\frac{1}{e} < 1$

(b) Verify that the function satisfies the hypotheses of the Mean Value Theorem on the given interval. Then find all numbers $c$ that satisfy the conclusion of the Mean Value Theorem.

$$f(x) = x^2 - 4x + 3, \ \ [1, 3]$$

5. (a) Find the local maximum and minimum values of $f(x) = x - 2\tan^{-1} x$ using the First Derivative Test.

(b) Find the intervals on which $f$ is concave up or concave down, and point(s) of inflection.

(c) Find the absolute maximum and absolute minimum values of $f$ on $[0, 4]$. **Hint:** $\tan^{-1}(4) = 1.325$.

6. The position of a particle is given by $s(t) = t^3 - 12t + 3$, $t \geq 0$, where $t$ is measured in seconds and $s$ in meters.

   (a) Find the velocity at time $t$ and after 2 seconds.

   (b) When is the particle at rest?

   (c) When is the particle moving forward?

   (d) Find the acceleration at time $t$ and after 2 seconds.

   (e) When is the particle speeding up? When is it slowing down?

WORKSPACE