

Please do not write in the boxes immediately below.

problem	1	2	3	4	5	6	total
points							

MATH 135 Fall 2023 Midterm Exam 2
November 16, 2023

Your name _____

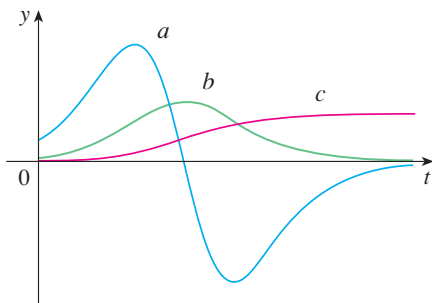
The exam has 6 different printed sides of exam problems and 1 side workspace.

Duration of the Midterm Exam is 90 minutes. There are 6 problems, worth 10 points each. From Problems 1 – 6, only 5 problems will be graded. If you solve all Problems 1 – 6, you must cross out the problem in the box above that must not be graded. If you solve all Problems 1 – 6 and do not cross out a problem, only the first five problems will be graded. Show all your work for full credit. Books, notes etc. are prohibited. Calculators, cellphones, earphones, AirPods and cheat sheets are NOT permitted.

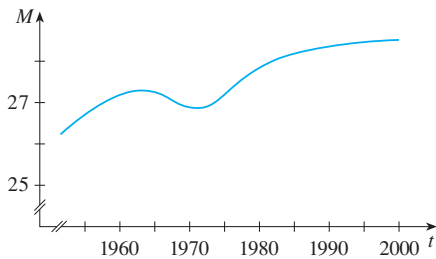
1. (i) The curve $y = \frac{1}{(1+x^2)}$ is called a **witch of Maria Agnesi**. Find an equation of the tangent line to this curve at the point $(-1, \frac{1}{2})$.

- (ii) Find a formula for $f^{(n)}(x)$ if $f(x) = \ln(x-1)$.

2. (i) The figure shows the graphs of three functions. One is the position function of a car, one is the velocity of the car, and one is its acceleration. Identify each curve, and explain your choices.



- (ii) The graph shows how the average age of first marriage of Japanese men varied in the last half of the 20th century. Sketch the graph of the derivative function $M'(t)$. During which years was the derivative negative?

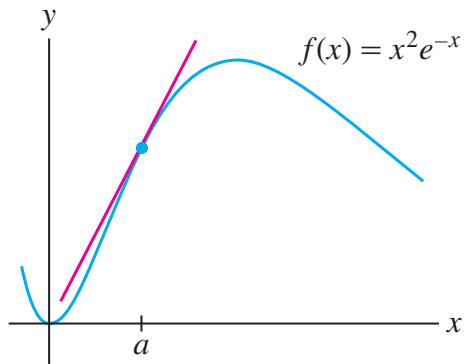


3. (i) Find all points on the graph of the function $f(x) = 2 \sin x + \sin^2 x$ at which the tangent line is horizontal.

- (ii) Compute $\frac{d}{dx} \csc^{-1}(e^x + 1)$ at $x = 0$.

4. (i) Find $(f^{-1})'(1)$ where $f(x) = x + \cos x$.

(ii) Find $a > 0$ such that the tangent line to the graph of $f(x) = x^2e^{-x}$ at $x = a$ passes through the origin.



5. (i) Use implicit differentiation to find dy/dx at the given point.

$$\sin(x + y) = y^2 \cos x, \quad \left(\frac{\pi}{2}, \frac{\pi}{2}\right)$$

- (ii) Use logarithmic differentiation to compute the derivative dy/dx

$$y = \left(\frac{(\tan^{-1} x) (e^{\tan x}) \sqrt{e^x + 1}}{e^{\sin x} \sec(2x)} \right)$$

6. (i) Find all critical points of the function $f(x) = x - \frac{4x}{x+1}$.

(ii) Find the absolute maximum and absolute minimum values of the function f in part (i) on $[0, 3]$.
Mention any theorem used.

(iii) Show that 5 is a critical number (point) of the function

$$g(x) = 2 + (x - 5)^3$$

but g does not have a local extreme value at 5.

WORKSPACE