Please do not write in the boxes immediately below.

problem	1	2	3	4	5	6	*EC*	total
points								

MATH 136 Fall 2023 Midterm Exam

October 5, 2023

Your name_____

The exam has 7 different printed sides of exam problems and 1 side workspace.

Duration of the Midterm Exam is 90 minutes. There are 6 problems, worth 10 points each, and an extra credit problem, worth 2 points. From Problems 1 – 6, only 5 problems will be graded. If you solve all Problems 1 – 6, you must cross out the problem in the box above that must not be graded. If you solve all Problems 1 – 6 and do not cross out a problem, only the first five problems will be graded. Show all your work for full credit. Books, notes etc. are prohibited. Calculators, cellphones, earphones, AirPods and cheat sheets are NOT permitted.

1) Draw a graph of the signed area represented by the integral and compute it using geometry. You are not allowed to use properties of integrals.

(i) $\int_{-5}^{5} x - 5 \, dx$

(iii) $\int_{-6}^{6} (|x| - 3) \, dx$

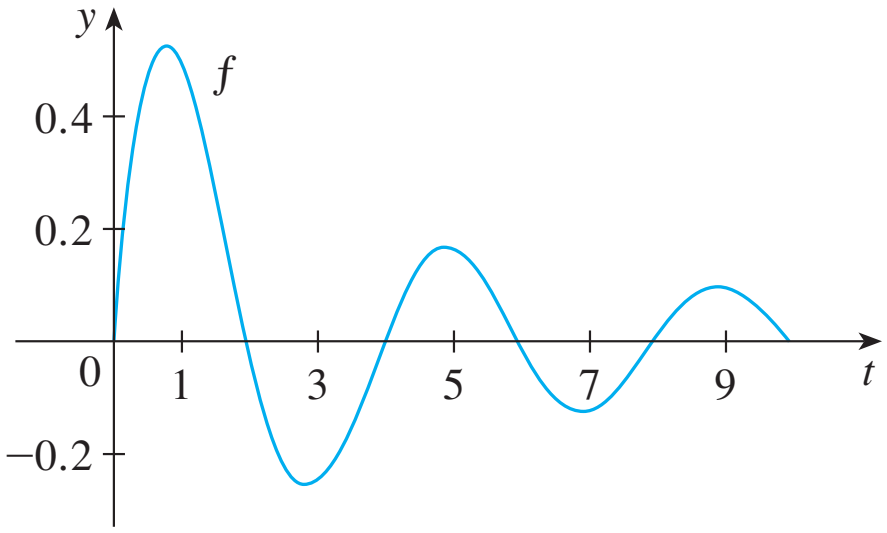
(ii) $\int_{-2}^{3} 4 \, dx$

(iv) $\int_{0}^{2} \sqrt{4 - x^2} + 2 \ dx$

2) Let A be the signed area between the graph of $f(x) = x^2 - 2x - 5$ and the x-axis from $x = 2$ to $x = 6$. Compute A as the limit $\lim_{n\to\infty} R_n$.

Hint: $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$, $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

3) Let $g(x) = \int_0^x f(t)\,dt$, where f is the function whose graph is shown.



(i) At what values of x do the local maximum and minimum values of g occur?

(ii) Where does t attain its absolute maximum value?

(iii) On what intervals is g concave downward?

(iv) Sketch the graph of g.

4) A particle moves along a line so that its velocity at time t is $v(t) = t^2 - 6t + 8$ (measured in meters per second).

(i) Find the displacement of the particle during the time period $0 \leq t \leq 4$.

(ii) **Set up** an integral to find the distance traveled by the particle during this time period. In your answer, there must be no absolute value functions, and the limits must be included in the integrals.

5) Evaluate the integrals using a suitable substitution.

(i) $\displaystyle\int \frac{x}{\sqrt{1+x}}\,dx$

(ii) $\displaystyle\int \frac{\sin 2x}{1+\sin^2 x}\,dx$ **Hint:** $\sin 2x = 2\sin x \cos x$

6) Evaluate the integrals.

(i) $\displaystyle\int \frac{2y^2 - 5y + 7\sqrt{y} + 3}{\sqrt{y}}\, dy$

(iii) $\displaystyle\int 3^{2x+1}\, dx$

(ii) $\displaystyle\int_0^{\pi/5} \sec 5x \tan 5x\ dx$

(iv) $\displaystyle\int_{\pi/8}^{\pi/6} \frac{\sec^2\theta - \tan^2\theta}{\cos^2(2\theta)}\, d\theta,$ **Hint:** $1 + \tan^2\theta = \sec^2\theta$

EXTRA CREDIT PROBLEM Use a substitution first and then use integration by parts to evaluate the integral.

$$\int \frac{x \tan^{-1}(\sqrt{x^2+1})}{\sqrt{x^2+1}}\, dx$$

WORKSPACE