Please do not write in the boxes immediately below.

problem	1	2	3	4	5	6	*EC*	total
points								

MATH 135 Fall 2023 Midterm Exam

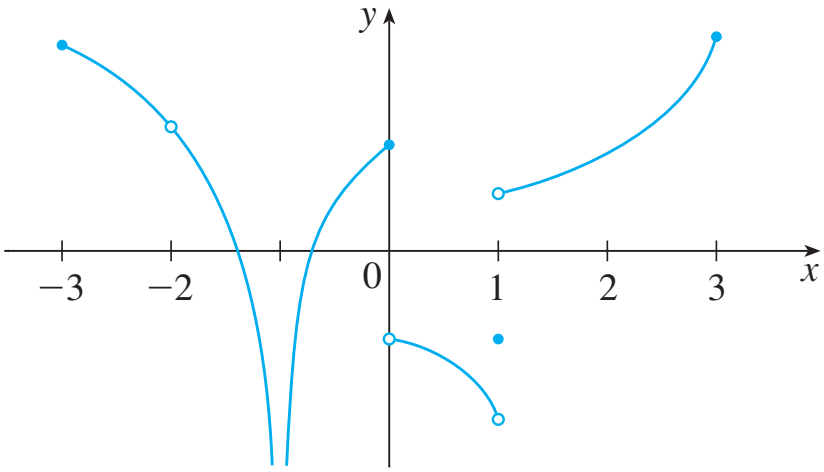
October 5, 2023

Your name_____

The exam has 7 different printed sides of exam problems and 1 side workspace.

Duration of the Midterm Exam is 90 minutes. There are 6 problems, worth 10 points each, and an extra credit problem, worth 2 points. From Problems 1 - 6, only 5 problems will be graded. If you solve all Problems 1 - 6, you must cross out the problem in the box above that must not be graded. If you solve all Problems 1 - 6 and do not cross out a problem, only the first five problems will be graded. Show all your work for full credit. Books, notes etc. are prohibited. Calculators, cellphones, earphones, AirPods and cheat sheets are NOT permitted.

1) For the function f whose graph is shown, state the following.



(i) $\lim_{x\to -3^+} f(x)$

(ii) $\lim_{x\to -2} f(x)$

(iii) $\lim_{x\to -1} f(x)$

(iv) $\lim_{x\to 0^-} f(x)$

(v) $\lim_{x\to 0^+} f(x)$

(vi) $\lim_{x\to 0} f(x)$

(vii) $\lim_{x\to 1} f(x)$

(viii) $f(1)$

(ix) The equations of the vertical asymptotes.

(x) The intervals on which f is continuos.

2) Evaluate the limits.

(i) $\lim_{x \to -1} \dfrac{3x^2 + 4x + 1}{x + 1}$

(ii) $\lim_{x \to 1} \dfrac{\frac{1}{x} - 1}{x - 1}$

(iii) $\lim_{x \to 0} \dfrac{\cos x - 1}{\sin x}$ **Hint:** $\sin^2 x + \cos^2 x = 1$

(iv) $\lim_{x \to 0} \dfrac{|x|}{x}$

3)

(i) Compute the limit

$$\lim_{x \to 0} \frac{\sin 3x}{\tan 2x}$$

(ii) Compute the limit

$$\lim_{h \to 0} \frac{\sin 4h - \sin 4h \cos 4h}{h^2}$$

4)

(i) Compute the limit using the Squeeze Theorem.

$$\lim_{x \to 0} \sin x \, \sin\left(\cos \frac{1}{x}\right)$$

(ii) Evaluate the limit

$$\lim_{x \to -\infty} \frac{4x - 3}{\sqrt{25x^2 + 4x}}$$

5)

(i) Show that there is a root of the equation

$$\sin x \cos x = \cos 2x$$

between 0 and $\pi/2$. Mention any theorem used.

(ii) Where is the function

$$f(x) = \begin{cases} \dfrac{x^2 - x}{x^2 - 1} & \text{if } x \neq 1 \\ 1 & \text{if } x = 1 \end{cases}$$

continuous?

6) Find an equation of the tangent line and normal line to the graph of $f(x) = \sqrt{x+5}$ at the point $(-1, 2)$.

EXTRA CREDIT PROBLEM Show that the function

$$f(x) = \begin{cases} \dfrac{x^2 - 16}{x - 4} & \text{if } x \neq 4 \\ 10 & \text{if } x = 4 \end{cases}$$

has a removable discontinuity at $x = 4$.

WORKSPACE