MATH 244 Linear Algebra

(1) Find the determinant of the following matrices.

(a)
$$\begin{bmatrix} 1 & 2 & -1 \\ 4 & -3 & 5 \\ 2 & 5 & 7 \end{bmatrix}$$
 (c) $\begin{bmatrix} 1 & 2 & -1 \\ 4 & 0 & 5 \\ 2 & 0 & 7 \end{bmatrix}$

(b)
$$\begin{bmatrix} 1 & 0 & 0 \\ 4 & -3 & 5 \\ 2 & 5 & 7 \end{bmatrix}$$
 (d) $\begin{bmatrix} 1 & 2 & -1 \\ 4 & -3 & 5 \\ 2 & 0 & 0 \end{bmatrix}$

		a	b	c	
(2)	Find the determinant of the upper triangular matrix	0	d	e	
		0	0	f	

(3) For which values of the scalar λ (lambda) is the matrix $\begin{bmatrix} \lambda & 1 & 1 \\ 1 & \lambda & -1 \\ 1 & 1 & \lambda \end{bmatrix}$ invertible?

- (4) For three column vectors $\vec{u}, \vec{v}, \vec{w}$ in \mathbb{R}^3 , what is the relationship between the determinants of $A = [\vec{u} \ \vec{v} \ \vec{w}]$ and $B = [\vec{u} \ \vec{w} \ \vec{v}]$? Note that matrix B is obtained by swapping the two columns of A.
- (5) Find the classical adjoint of the matrix and use the result to find A^{-1} .

	[1	0	1]
(a)	0	1	0
	2	0	1



(6) Calculate the area of the parallelogram spanned by $\vec{u} = \begin{bmatrix} 1\\0\\3 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} 2\\1\\1 \end{bmatrix}$

(7) Compute the volume of the parallelepiped spanned by $\vec{u} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \ \vec{v} = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}, \ \vec{w} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}.$

(8) Find the volume of the parallelepiped spanned by \vec{u}, \vec{v} , and \vec{w} in the following figure.



(9) Find two unit vectors perpendicular to both $\vec{a} = \begin{bmatrix} 3\\1\\1 \end{bmatrix}$, $\vec{b} = \begin{bmatrix} -1\\2\\1 \end{bmatrix}$.

(10) Use the cross product to find the area of the triangle with vertices P = (1, 1, 5), Q = (3, 4, 3), and R = (1, 5, 7).

