MATH 244 Linear Algebra

Determine whether the statements that follow are true or false, and give a specific counterexample when false.

- (1) If columns 1 and 3 of B are the same, so are columns 1 and 3 of AB
- (2) If rows 1 and 3 of B are the same, so are rows 1 and 3 of AB.
- (3) If rows 1 and 3 of A are the same, so are rows 1 and 3 of AB.
- (4) $(AB)^2 = A^2 B^2$
- (5) If A^2 is defined then A is necessarily square.
- (6) If AB and BA are defined then A and B are square.
- (7) If AB and BA are defined then AB and BA are square.
- (8) If AB = B then A = I.
- (9) A 4 by 4 matrix with a row of zeros is not invertible.
- (10) A matrix with 1s down the main diagonal is invertible
- (11) If A is invertible then A^{-1} is invertible.
- (12) f A^T is invertible then A is invertible.
- (13) If A is invertible and its rows are in reverse order in B, then B is invertible.
- (14) If A and B are symmetric then AB is symmetric.
- (15) If A and B are invertible then BA is invertible
- (16) Every nonsingular matrix can be factored into the product A = LU of a lower triangular L and an upper triangular U
- (17) If all diagonal entries of A are zero, then A is singular.
- (18) If $A^2 + A = I$ then $A^{-1} = A + I$
- (19) If all diagonal entries of A are zero, then A is singular.
- (20) A square matrix has no free variables.
- (21) An invertible matrix has no free variables.
- (22) An m by n matrix has no more than n pivot variables.
- (23) An m by n matrix has no more than m pivot variables.

Defintion A *nonsingular* matrix is a square matrix that is invertible. A square matrix that is not invertible is called a *singular* matrix.