MATH 244 Linear Algebra

(1) Compute the dot product

(a)
$$\begin{bmatrix} 1 \ 2 \ 3 \ 4 \end{bmatrix} \cdot \begin{bmatrix} 5 \\ 6 \\ 7 \\ 8 \end{bmatrix}$$
 (b) $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$

(2) Is the vector
$$\hat{b} = \begin{bmatrix} 1\\1\\1 \end{bmatrix}$$
 a linear combination of the vectors $\hat{v} = \begin{bmatrix} 1\\2\\3 \end{bmatrix}$ and $\hat{w} = \begin{bmatrix} 4\\5\\6 \end{bmatrix}$?

(3) Find all vectors in \mathbb{R}^3 perpendicular to $\begin{bmatrix} 1\\3\\-1 \end{bmatrix}$.

(4) For which values of a, b, c, d, and e is the following matrix in reduced row-echelon form?

$$A = \begin{bmatrix} 0 & a & 2 & 1 & b \\ 0 & 0 & 0 & c & d \\ 0 & 0 & e & 0 & 0 \end{bmatrix}$$

- (5) Find all 4×1 matrices in reduced row-echelon form.
- (6) Balancing a chemical reaction. Consider the chemical reaction

$$a \operatorname{NO}_2 + b \operatorname{H}_2 \operatorname{O} \rightarrow c \operatorname{HNO}_2 + d \operatorname{HNO}_3,$$

where a, b, c, and d are unknown positive integers. The reaction must be balanced; that is, the number of atoms of each element must be the same before and after the reaction. For example, because the number of oxygen atoms must remain the same,

$$2a + b = 2c + 3d.$$

While there are many possible values for a, b, c, and d that balance the reaction, it is customary to use the smallest possible positive integers. Balance this reaction.

- (7) Compute the product two ways: in terms of the columns of A and in terms of the rows of A.
 - $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$

(8) Find a 3×3 matrix A such that

$$A \begin{bmatrix} 1\\0\\0 \end{bmatrix} = \begin{bmatrix} 1\\2\\3 \end{bmatrix}, A \begin{bmatrix} 0\\1\\0 \end{bmatrix} = \begin{bmatrix} 4\\5\\6 \end{bmatrix}, \text{ and } A \begin{bmatrix} 0\\0\\1 \end{bmatrix} = \begin{bmatrix} 7\\8\\9 \end{bmatrix}.$$

(9) For which values of the constants b and c is the vector $\begin{bmatrix} 3\\b\\c \end{bmatrix}$ a linear combination of the vectors $\begin{bmatrix} 1\\3\\2 \end{bmatrix}$, $\begin{bmatrix} 2\\6\\4 \end{bmatrix}$ and $\begin{bmatrix} -1\\-3\\-2 \end{bmatrix}$?

(10) For which values of the constant
$$c$$
 is $\begin{bmatrix} 1 \\ c \\ c^2 \end{bmatrix}$ a linear combination of $\begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 3 \\ 9 \end{bmatrix}$?