Your Name:

Duration of the Quiz is 50 minutes. There are two problems, worth 20 points. Show all your work for full credit. Books, notes etc. are prohibited.

(1) Consider the linear mapping $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined by $A = \begin{bmatrix} 0 & 3 & 0 \\ 1 & 0 & -1 \\ 0 & 2 & 0 \end{bmatrix}$.

Determine whether the mapping is diagonalizable. If it is, find a basis (eigenbasis) of the appropriate vector space consisting of eigenvectors, and diagonalize A.

(2) Let W be the subspace of $\mathbb{R}^4$ spanned by $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix}$.

(a) Verify that $\mathbf{v}_1, \mathbf{v}_2$ and $\mathbf{v}_3$ are pairwise orthogonal.

(b) Find a basis for $W^{\perp}$.

(c) Find an orthonormal basis of $\mathbb{R}^4$ based on your answers to part (a) and part (b).

(d) Find $\mathrm{proj}_W(\mathbf{x})$ for $\mathbf{x} = \begin{bmatrix} 1 \\ 3 \\ 1 \\ 7 \end{bmatrix}$.

(e) Can you explain your answer to part (b) in terms of the mapping $\mathrm{proj}_W(\cdot)$?