STAT 244 Linear Algebra **Quiz 6** **April 11, 2025**

Your Name:

Duration of the Quiz is 20 minutes. There are three problems, worth 20 points. Show all your work for full credit. Books, notes etc. are prohibited.

(1) Let $V = \mathbb{R}^2$ and $W = P_3(\mathbb{R})$. If T is a linear transformation that satisfies $T(1,1) = x + x^2$ and $T(3,0) = x - x^3$, what is $T(2,2)$?

(2) Let $V = \mathbb{R}^3$ and $W = \mathbb{R}^4$, and let $T : V \to W$ be defined by

$$T(x_1, x_2, x_3) = (x_1 - x_2, x_2 - x_3, x_1 + x_2 - x_3, x_3 - x_1).$$

What is the matrix of T with respect to the standard bases in V and W?

(3) For the following matrix, defining linear maps T between vector spaces of the appropriate dimensions, find bases for $\mathrm{Ket}(T)$ and $\mathrm{Im}(T)$. Also, determine whether T is injective or surjective, both, or neither.

$$\begin{bmatrix} 1 & 0 & 1 & -1 & 0 & 1 \\ -1 & 1 & 2 & 1 & 1 & 0 \\ 0 & 1 & 3 & 2 & 2 & 0 \end{bmatrix}$$