STAT 244 Linear Algebra **Quiz 5** **March 28, 2025**

Your Name:

Duration of the Quiz is 25 minutes. There are five problems, worth 20 points. Show all your work for full credit. Books, notes etc. are prohibited.

(1) Prove that the set $\{(1,1,1),(1,1,0),(1,0,0)\}$ is a basis of $\mathbb{R}^3$.

(2) Prove that the set $\{(x,x,x,y)\,|\,x,y\in\mathbb{R}\}$ is a subspace of $\mathbb{R}^4$, and find its dimension.

(3) Find a basis of the kernel of the following matrix:

$$\begin{bmatrix} 1 & 0 & 5 & 0 \\ 0 & 1 & 7 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(4) Is $\{1+x, 2-x^2, 3+5x^2, 7-2x\}$ a linearly independent subset of $\mathcal{P}_2(\mathbb{R})$? Justify your answer.

(5) Let $W_1 = \{(a,0,0) \,|\, a \in \mathbb{R}\}$, $W_2 = \{(0,b,0) \,|\, b \in \mathbb{R}\}$, and $W_3 = \{(0,0,c) \,|\, c \in \mathbb{R}\}$. Find $W_1 + W_2 + W_3$. Give a reason.