Your Name:

Duration of the Quiz is 25 minutes. There are two problems, worth 20 points. Show all your work for full credit. Books, notes etc. are prohibited.

(1) (2 points each) Determine whether the statements that follow are true or false, and justify your answer.

(a) If A is a 3×4 matrix of rank 3, then the system $A\vec{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ must have infinitely many solutions.

(b) If two matrices A and B have the same reduced row-echelon form, then the equations $A\vec{x} = \vec{0}$ and $B\vec{x} = \vec{0}$ must have the same solutions.

(c) The linear system $A\vec{x} = \vec{b}$ is consistent if and only if $\text{rank}(A) = \text{rank}\,[\,A \,|\, \vec{b}\,]$.

(d) If A and B are matrices of the same size, then the formula $\text{rank}(A+B) = \text{rank}(A) + \text{rank}(B)$ must hold.

(e) If vector $\vec{w}$ is a linear combination of $\vec{u}$ and $\vec{v}$, then $\vec{u} + \vec{v} + \vec{w}$ must be a linear combination of $\vec{u}$ and $\vec{u} + \vec{v}$.

(f) If the system $A\vec{x} = \vec{b}$ has a unique solution, then A must be a square matrix.

(2) It is known that the vector $\begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$ can be uniquely written as a linear combination of the vectors $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$.

That is, there exist unique scalars a, b, and c such that $\begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} = a \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + b \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} + c \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$.

(a) (1 point) Write down a system of equations to find a, b, and c.

(b) (1 point) Write the system in part (a) in matrix form.

(c) (4 points) Find the inverse of the coefficient matrix using Gauss-Jordan method.

(d) (2 points) Solve the system using the inverse of the coefficient matrix.