## Your Name:

Duration of the Quiz is 25 minutes. There are three problems, worth 20 points. Show all your work for full credit. Books, notes etc. are prohibited.

- (1) (2 points each) Determine whether the statements that follow are true or false, and justify your answer.
  - (i) There exists a  $3 \times 4$  matrix with rank 4.
  - (ii) There exists a system of three linear equations with three unknowns that has exactly three solutions.
  - (iii) A linear system with fewer unknowns than equations must have infinitely many solutions or none.
  - (iv) A lower triangular  $3 \times 3$  matrix has rank 3 if (and only if) the product of its diagonal entries is nonzero.
  - (v) A matrix is called a 0-1-matrix if all of its entries are ones and zeros. The majority of the 0-1-matrices of size  $3 \times 3$  have rank 3.
  - (vi) There exists scalars a and b such that matrix  $\begin{bmatrix} 0 & 1 & a \\ -1 & 0 & b \\ -a & -b & 0 \end{bmatrix}$  has rank 3.

(2) Compute the following products.

(a) 
$$\begin{bmatrix} 1 & -2 & 3 \\ 2 & -3 & 5 \\ -1 & 7 & -9 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -1 & 3 \\ -3 & 5 \end{bmatrix}$$
 (b)  $\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} \begin{bmatrix} 0 & 1 & -2 \end{bmatrix}$ 

(3) Find all vectors in  $\mathbb{R}^3$  perpendicular to  $\begin{bmatrix} 1\\ 2\\ -1 \end{bmatrix}$ .