Due by 4pm on Friday, March 21. Please leave your homework on the table before class begins on Friday or leave it in the dropbox outside my office. Do not forget to attach the honor code. There are five problems, 20 points each.

(1) (a) Let $S = \{(1,0,0),(0,0,2)\}$ in $\mathbb{R}^3$. Which vectors are in Span(S)? Describe this geometrically.

(b) Same question for $S = \{(1,4,0,0),(2,3,0,0)\}$ in $\mathbb{R}^4$.

(c) Same question for $S = \{(1,1,1)\}$ in $\mathbb{R}^3$.

(d) Same question for $S = \{1, x, x^2\}$ in $\mathcal{P}_4(\mathbb{R})$.

(2) (a) In $V = C(\mathbb{R})$, let $S_1 = \{\sin x, \cos x, \sin^2 x, \cos^2 x\}$ and $S_2 = \{1, \sin(2x), \cos(2x)\}$. Is Span$(S_1) =$ Span(S_2)? Why or why not?

(b) In $V = \mathcal{P}_2(\mathbb{R})$, let $S = \{1, 1+x, 1+x+x^2\}$. Show that Span$(S) = \mathcal{P}_2(\mathbb{R})$.

(c) Let $S = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \right\}$ in $M_{2\times 2}(\mathbb{R})$. Describe the subspace Span(S).

(d) Same question for $S = \left\{ \begin{bmatrix} 2 & -1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ -1 & 1 \end{bmatrix} \right\}$.

(3) (a) Show that in $V = \mathbb{R}^2$, any set of three or more vectors is linearly dependent.

(b) What is the analogous statement for $V = \mathbb{R}^3$? Prove your assertion.

(c) Show that $S = \left\{ \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 3 \\ 2 & 1 \end{bmatrix} \right\}$ is a linearly independent subset of $M_{2\times 2}(\mathbb{R})$.

(d) Show that $S = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$ is a linearly independent subset of $M_{2\times 2}(\mathbb{R})$ and that Span$(S) = M_{2\times 2}(\mathbb{R})$.

(4) (a) Let $\mathbf{v}, \mathbf{w} \in V$. Show that $\{\mathbf{v}, \mathbf{w}\}$ is linearly dependent if and only if $\mathbf{v}$ is a scalar multiple of $\mathbf{w}$, or $\mathbf{w}$ is a scalar multiple of $\mathbf{v}$.

(b) Show by example, however, that there are linearly dependent sets of three vectors such that no pair are scalar multiples of each other.

(c) Let $\mathbf{v}, \mathbf{w} \in V$. Show that $\{\mathbf{v}, \mathbf{w}\}$ is linearly independent if and only if $\{\mathbf{v}+\mathbf{w}, \mathbf{v}-\mathbf{w}\}$ is linearly independent.

(d) Let $\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}, \ldots, \mathbf{v_n} \in V$. Show that if $\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}, \ldots, \mathbf{v_n}\}$ is linearly independent, then

$$\{\mathbf{v_1}, \mathbf{v_1}+\mathbf{v_2}, \mathbf{v_1}+\mathbf{v_2}+\mathbf{v_3}, \ldots, \mathbf{v_1}+\mathbf{v_2}+\mathbf{v_3}+\ldots+\mathbf{v_n}\}$$

is linearly independent.

(5) Determine whether each of the following sets of vectors is linearly dependent or linearly independent:

(a) $S = \{(1,1),(1,3),(0,2)\}$ in $\mathbb{R}^2$.

(b) $S = \{(1,2,1),(1,3,0)\}$ in $\mathbb{R}^3$.

(c) $S = \{(1,1,1,1),(1,0,0,1),(3,0,0,0)\}$ in $\mathbb{R}^4$.

(d) $S = \{x^4, x^4+x^3, x^4+x^3+x^2\}$ in $\mathcal{P}_4(\mathbb{R})$.

(e) $S = \{e^x, \cos x\}$ in $C(\mathbb{R})$.