Due by 4pm on Friday, February 7. Please leave your homework on the table before class begins on Friday or leave it in the dropbox outside my office. Do not forget to attach the honor code. Each problem is worth 20 points.

(1) (a) Find all solutions of the equations using Gauss-Jordan elimination. Show all your work.

$$\begin{aligned} 4x_1 + 3x_2 + 2x_3 - x_4 &= 4 \\ 5x_1 + 4x_2 + 3x_3 - x_4 &= 4 \\ -2x_1 - 2x_2 - x_3 + 2x_4 &= -3 \\ 11x_1 + 6x_2 + 4x_3 + x_4 &= 11 \end{aligned}$$

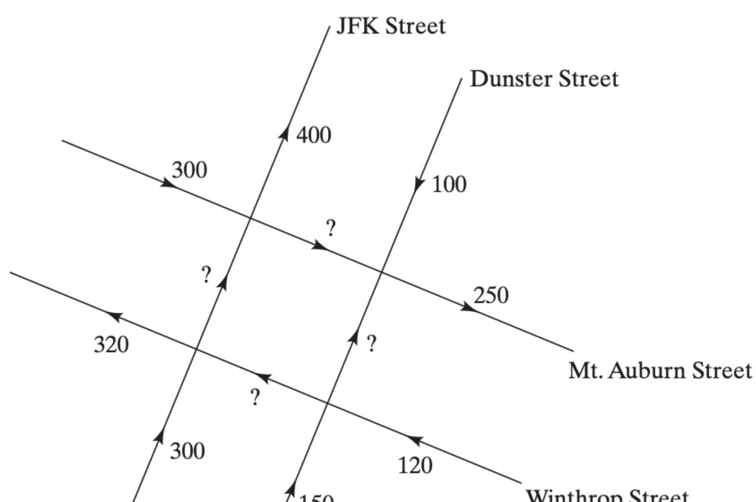
(b) Find the polynomial of degree 4 whose graph goes through the points $(1, 1)$, $(2, -1)$, $(3, -59)$, $(-1, 5)$ and $(-2, -29)$. Graph this polynomial.

(2) (a) How many types of 2×2 matrices in reduced row-echelon form are there?
(b) How many types of 3×2 matrices in reduced row- echelon form are there?

(3) (a) Suppose matrix A is transformed into matrix B by a sequence of elementary row operations. Is there a sequence of elementary row operations that transforms B into A? Explain your answer.

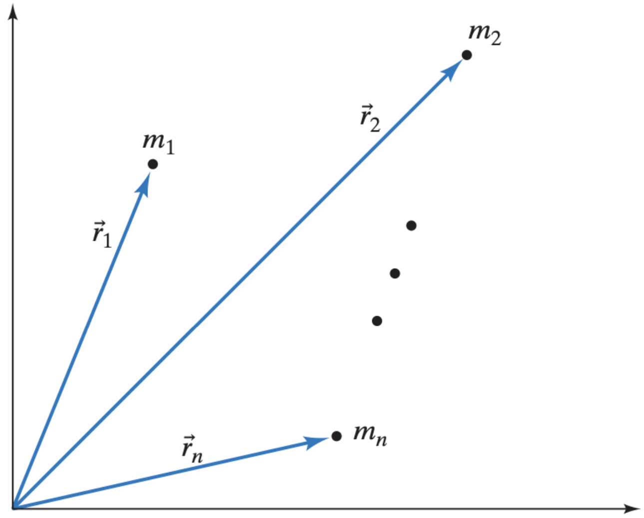
(b) Is there a sequence of elementary row operations that transforms $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ into $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$? Explain.

(4) The accompanying sketch represents a maze of one-way streets in a city in the United States. The traffic volume through certain blocks during an hour has been measured. Suppose that the vehicles leaving the area during this hour were exactly the same as those entering it.



What can you say about the traffic volume at the four locations indicated by a question mark? Can you figure out exactly how much traffic there was on each block? If not, describe one possible scenario. For each of the four locations, find the highest and the lowest possible traffic volume.

(5) Consider some particles in the plane with position vectors $\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_n$ and masses $m_1, m_2, \ldots, m_n$.



The position vector of the *center of mass* of this system is

$$\vec{r}_{cm} = \frac{1}{M}(m_1\vec{r}_1 + m_2\vec{r}_2 + \cdots + m_n\vec{r}_n),$$

where $M = m_1 + m_2 + \cdots + m_n$.

Consider the triangular plate shown in the accompanying sketch. How must a total mass of 1 kg be distributed among the three vertices of the plate so that the plate can be supported at the point $\begin{bmatrix} 2 \\ 2 \end{bmatrix}$; that is, $\vec{r}_{cm} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$? Assume that the mass of the plate itself is negligible.