Due by 4pm on Wednesday, April 30. Please leave your homework on the table before class begins on Friday or leave it in the dropbox outside my office. Do not forget to attach the honor code.

(1) (5 points each) Compute the characteristic polynomials of the following mappings:

(a) $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined by $A = \begin{bmatrix} 1 & -2 & 1 \\ 1 & -2 & 1 \\ 1 & -2 & 1 \end{bmatrix}$.

(b) $T : \mathcal{P}_3(\mathbb{R}) \to \mathcal{P}_3(\mathbb{R})$ defined by $T(p) = xp' - 4p$.

(2) (10 points each) For each of the following linear mappings:

- Find all the eigenvalues, and
- For each eigenvalue λ, find a basis of the eigenspace E_λ.

(a) $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined by $A = \begin{bmatrix} 1 & -1 & 3 \\ 0 & 1 & 0 \\ 0 & 4 & 1 \end{bmatrix}$.

(b) $T : \mathcal{P}_3(\mathbb{R}) \to \mathcal{P}_3(\mathbb{R})$ defined by $T(p(x)) = p'(x) + 2p(x)$.

(c) $V = \mathrm{Span}(e^x, xe^x, e^{2x}, xe^{2x})$, $T : V \to V$ defined by $T(f) = f''$.

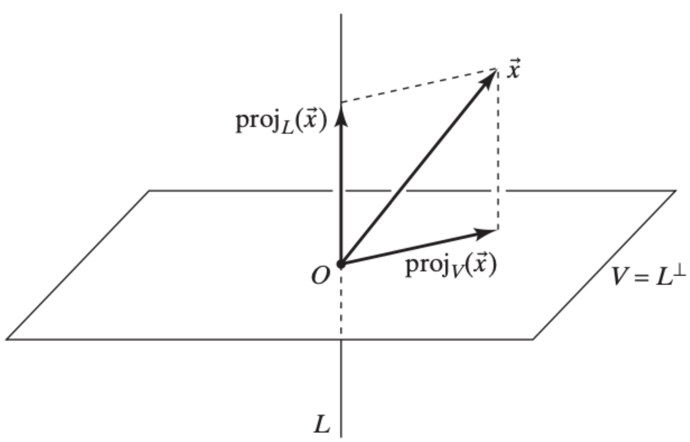
(3) (5 points each) Let $A \in M_{n\times n}(\mathbb{R})$, and assume that $\det(A - \lambda I)$ factors completely into $(\lambda - \lambda_1)(\lambda - \lambda_2)\cdots(\lambda - \lambda_n)$, a product of linear factors.

(a) Trace of A, denoted $\mathrm{Tr}(A)$, is defined to be the sum of the diagonal entries of A. Show that

$$\mathrm{Tr}(A) = \lambda_1 + \lambda_2 + \cdots + \lambda_n.$$

(b) Show that $\det(A) = \lambda_1\lambda_2\cdots\lambda_n$.

(4) (20 points) Let V be a plane in $\mathbb{R}^3$ that goes through the origin, and L a line perpendicular to V that goes through the origin. Let $\mathbf{x} \in \mathbb{R}^3$. Use the following diagram to find formulas for the orthogonal projection of $\mathbf{x}$ onto V, $\mathrm{proj}_V(\mathbf{x})$, as well as for the reflection of $\mathbf{x}$ about V, $\mathrm{ref}_V(\mathbf{x})$.



Let V be the plane defined by $2x_1 + x_2 - 2x_3 = 0$, and let $\mathbf{x} = \begin{bmatrix} 5 \\ 4 \\ -2 \end{bmatrix}$. Find $\mathrm{ref}_V(\mathbf{x})$.

(5) (10 points each) Determine whether the given linear mapping is diagonalizable. If it is, find a basis of the appropriate vector space consisting of eigenvectors. Such a basis is called an *eigenbasis*.

(a) $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined by $A = \begin{bmatrix} 0 & 3 & 0 \\ 1 & 0 & -1 \\ 0 & 2 & 0 \end{bmatrix}$.

(b) $T : \mathbb{R}^4 \to \mathbb{R}^4$ defined by $A = \begin{bmatrix} 4 & 2 & 7 & -1 \\ -1 & 1 & 3 & 2 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$.

(6) Let $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 1 \\ 0 & 1 & 3 \end{bmatrix}$. Find A^{-1} using the characteristic polynomial of A, $p(t) = \det(A - tI)$.