Due by 4pm on Friday, January 31. Please leave your homework on the table before class begins on Friday or leave it in the dropbox outside my office. Do not forget to attach the honor code. Each problem is worth 20 points.

(1) Determine whether the system of linear equations is consistent or inconsistent. If consistent, find all solutions.

(a) $x + 2y + 3z = 1$
$x + 3y + 4z = 3$
$x + 4y + 5z = 4$

(b) $x + 2y + 3z = 1$
$3x + 2y + z = 1$
$7x + 2y - 3z = 1$

(c) $x + y - z = 0$
$4x - y + 5z = 0$
$6x + y + 4z = 0$

(2) Consider the linear system

$$\begin{aligned} x + y - z &= -2 \\ 3x - 5y + 13z &= 18 \\ x - 2y + 5z &= k \end{aligned}$$

where k is an arbitrary number.

(a) For which value(s) of k does this system have one or infinitely many solutions?
(b) For each value of k you found in part a, how many solutions does the system have?
(c) Find all solutions for each value of k.

(3) Find all polynomials $f(t)$ of degree ≤ 2 (of the form $f(t) = a + bt + ct^2$) whose graphs run through the points $(1, 3)$ and $(2, 6)$, such that $f'(1) = 1$, where $f'(t)$ denotes the derivative.

(4) Find a system of linear equations with three unknowns whose solutions are the points on the line through $(1, 1, 1)$ and $(3, 5, 0)$.

(5) In a grid of wires, the temperature at exterior mesh points is maintained at constant values (in $°C$), as shown in the accompanying figure. When the grid is in thermal equilibrium, the temperature T at each interior mesh point is the average of the temperatures at the four adjacent points. For example,

$$T_2 = \frac{T_3 + T_1 + 200 + 0}{4}.$$

Find the temperatures T_1, T_2, and T_3 when the grid is in thermal equilibrium.