(1) For each of the following linear mappings:

- Find all the eigenvalues, and
- For each eigenvalue λ, find a basis of the eigenspace E_λ.

(a) $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined by $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$.

(b) $T : \mathcal{P}_3(\mathbb{R}) \to \mathcal{P}_3(\mathbb{R})$ defined by $T(p(x)) = 2p'(x) + 3p(x)$

(2) Determine whether the given linear mapping is diagonalizable. If it is, find a basis of the appropriate vector space consisting of eigenvectors. Such a basis is called an *eigenbasis*.

(a) $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $A = \begin{bmatrix} 3 & 1 \\ -1 & 1 \end{bmatrix}$.

(b) $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined by $A = \begin{bmatrix} 2 & 0 & 0 \\ 2 & 1 & 3 \\ 5 & 0 & 1 \end{bmatrix}$.

(3) Let $A = \begin{bmatrix} 1 & -1 & 3 \\ 0 & 1 & 0 \\ 0 & 4 & 1 \end{bmatrix}$. Find A^{-1} using the characteristic polynomial of A, $p(t) = \det(A - tI)$.