(1) Let $T : V \to W$ be a linear transformation from a four-dimensional vector space V with basis $\alpha = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ to a three-dimensional vector space W with basis $\beta = \{\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3\}$ with matrix

$$[T]_\alpha^\beta = \begin{bmatrix} 1 & 0 & 1 & 2 \\ 2 & 1 & 0 & 1 \\ 1 & -1 & 3 & 5 \end{bmatrix}$$

Find the dimension of $\ker(T)$ and dimension of $\mathrm{Im}(T)$.

(2) Let $T : V \to W$ be a linear transformation from a four-dimensional vector space V with basis $\alpha = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ to a two-dimensional vector space W with basis $\beta = \{\mathbf{w}_1, \mathbf{w}_2\}$ with matrix

$$[T]_\alpha^\beta = \begin{bmatrix} 1 & 2 & 0 & 1 \\ 1 & 2 & 1 & 0 \end{bmatrix}$$

Find a basis for V so that the first $\dim \ker(T)$ members are in $\ker(T)$.