

Due by 9am on December 4. Please upload your solutions to Canvas as one PDF file. Do not forget to attach the honor code. You must show all your work for full credit. Each problem is worth 10 points.

(1) Determine whether the sequence converges or diverges. If it converges, find the limit.

$$(a) a_n = \frac{4 + n - 3n^2}{4n^2 + 1}$$

$$(b) d_n = \ln(n^2 + 4) - \ln(n^2 - 1)$$

$$(c) c_n = \frac{(-1)^n}{\sqrt{n}}$$

$$(d) b_n = \tan^{-1}\left(1 - \frac{2}{n}\right)$$

(2) Determine whether the sequence converges or diverges. If it converges, find the limit.

$$(a) a_n = \sqrt{n} \ln\left(1 + \frac{1}{n}\right)$$

$$(b) a_n = n(\sqrt{n^2 + 1} - n)$$

(3) Determine whether the sequence converges or diverges. If it converges, find the limit.

$$(a) a_n = \frac{1}{\sqrt{n^4 + n^8}}$$

$$(b) a_n = (2^n + 3^n)^{1/n}$$

$$(c) a_n = (n + 10^n)^{1/n}$$

$$(d) a_n = 2^{-n} \cos n\pi$$

(4) Determine whether the sequence is increasing, decreasing, or not monotonic. Is the sequence bounded?

$$(a) a_n = \cos n$$

$$(b) a_n = 3 - 2ne^{-n}$$

$$(c) a_n = n^3 - 3n + 3$$

$$(d) a_n = 2 + \frac{(-1)^n}{n}$$

(5) Find the limit of the sequence $\{\sqrt{2}, \sqrt{2\sqrt{2}}, \sqrt{2\sqrt{2\sqrt{2}}}, \dots\}$.

(6) Determine whether the series is convergent or divergent by expressing S_n as a telescoping sum. If it is convergent, find its sum.

$$(a) \sum_{n=2}^{\infty} \frac{2}{n^2 - 1}$$

$$(b) \sum_{n=1}^{\infty} \ln \frac{n}{n+1}$$

$$(c) \sum_{n=1}^{\infty} (e^{1/n} - e^{1/(n+1)})$$

$$(d) \sum_{n=2}^{\infty} \frac{1}{n^3 - n}$$

(7) Determine whether the series is convergent or divergent. If it is convergent, find its sum.

$$(a) \sum_{n=1}^{\infty} \frac{1}{4 + e^{-n}}$$

$$(b) \sum_{k=1}^{\infty} \frac{k^2}{k^2 - 2k + 5}$$

$$(c) \sum_{n=1}^{\infty} \left(\frac{3}{5^n} + \frac{2}{n}\right)$$

$$(d) \sum_{n=1}^{\infty} \left(\frac{1}{e^n} + \frac{1}{n(n+1)}\right)$$

(8) Find the values of x for which the series converges. Find the sum of the series for those values of x .

$$(a) \sum_{n=1}^{\infty} (-5)^n x^n$$

$$(b) \sum_{n=0}^{\infty} \frac{(x-2)^n}{3^n}$$

$$(c) \sum_{n=1}^{\infty} (x+2)^n$$

$$(d) \sum_{n=0}^{\infty} (-4)^n (x-5)^n$$

(9) Use the Integral Test to determine whether the series is convergent or divergent.

$$(a) \sum_{n=1}^{\infty} \frac{n}{n^2 + 1}$$

$$(b) \sum_{n=1}^{\infty} \frac{2}{5n-1}$$

$$(c) \sum_{n=1}^{\infty} \frac{1}{(3n-1)^4}$$

$$(d) \sum_{n=0}^{\infty} n^2 e^{-n^3}$$

(10) Explain why the Integral Test can't be used to determine whether the series is convergent.

$$(a) \sum_{n=1}^{\infty} \frac{\cos \pi n}{\sqrt{n}}$$

$$(b) \sum_{n=1}^{\infty} \frac{\cos^2 n}{1 + n^2}$$