Due by 9am on December 4. Please upload your solutions to Canvas as one PDF file. Do not forget to attach the honor code. You must show all your work for full credit. Each problem is worth 10 points.

(1) (a) Find the absolute maximum and absolute minimum values of f on the given interval.

$$f(x) = \sin x \cos x, \ \left[0, \frac{\pi}{2}\right]$$

(b) Find a cubic function $f(x) = ax^3 + bx^2 + cx + d$ that has a local maximum value of 3 at $x = -2$ and a local minimum value of 0 at $x = 1$.

(2) Prove that $f(x) = x^5 + 2x^3 + 4x - 12$ has precisely one real root.

(3) Prove that $f(x) = x^4 + 5x^3 + 4x$ has no root c satisfying $c > 0$.

(4) Prove that $c = 4$ is the largest root of $f(x) = x^4 - 8x^2 - 128$.

(5) (a) Find a point c satisfying the conclusion of the MVT for the given function and interval.

$$f(x) = x - \sin(\pi x), \quad [-1, 1]$$

(b) Let $f(x) = 2 - |2x - 1|$. Show that there is no value of c such that $f(3) - f(0) = f'(c)(3 - 0)$. Why does this not contradict the Mean Value Theorem?

(6) Sam made two statements that Deborah found dubious.

(a) "The average velocity for my trip was 70 mph; at no point in time did my speedometer read 70 mph."
(b) "A policeman clocked me going 70 mph, but my speedometer never read 65 mph."

In each case, which theorem did Deborah apply to prove Sam's statement false: the Intermediate Value Theorem or the Mean Value Theorem? Explain.

(7) Find all critical points of f and use the First Derivative Test to determine whether they are local minima or maxima.

$$f(x) = x^3 - 12x - 4$$

(8) (a) Determine where the function $f(x) = (1000 - x)^2 + x^2$ is decreasing.
(b) **Use part (a)** to decide which is larger: $800^2 + 200^2$ or $600^2 + 400^2$.

(9) Let $f(x) = 2x^4 - 3x^2 + 2$. Determine the intervals on which the function is concave up or down and find the points of inflection.

(10) Find the local maximum and minimum values of f using both the First and Second Derivative Tests. Which method do you prefer?

$$f(x) = 1 + 3x^2 - 2x^3$$

(11) Suppose $f(3) = 2, f'(3) = \frac{1}{2}$, and $f'(x) > 0$ and $f''(x) < 0$ for all x.

(a) Sketch a possible graph for f.
(b) How many solutions does the equation $f(x) = 0$ have? why?
(c) Is it possible that $f'(2) = \frac{1}{3}$? Why?