Due by 5pm on Friday, December 15. Do not forget to attach the honor code. There are five problems.

(1) Let $g(x) = (1+x)(1+x+x^3) \in \mathbb{F}_2[x]$ be the generator polynomial of a binary $[7,3]$-cyclic code C. Write down a generator matrix and a parity-check matrix for C. Construct a generator matrix of the form $(I_3|A)$.

(2) Write down a parity-check matrix H for a binary Hamming code of length 15, where the jth column of H is the binary representation of j. Then use H to construct a syndrome look-up table and use it to decode the following words:
 (a) 01010 01010 01000,
 (b) 11100 01110 00111,
 (c) 11001 11001 11000.

(3) A binary $[15,7]$-cyclic code is generated by $g(x) = 1 + x^4 + x^6 + x^7 + x^8$. Decode the following received words using error trapping:
 (a) 110111101110110
 (b) 111110100001000

(4) A binary $[15,5]$-cyclic code is generated by $g(x) = 1 + x + x^2 + x^4 + x^5 + x^8 + x^{10}$. Construct a parity-check matrix of the form $(I_{10}|A)$.

(5) A binary $[15,5]$-cyclic code is generated by $g(x) = 1 + x + x^2 + x^4 + x^5 + x^8 + x^{10}$. Decode the following words using error trapping:
 (a) 011111110101000
 (b) 100101111011100