MATH 361 Real Analysis **Homework Assignment 6** **Spring 2024**

Due by 4pm on March 22. Do not forget to attach the honor code. Each problem is worth 10 points.

1. Suppose $a_n \geq 0$ for all n.
   (a) Prove that if $\lim a_n = 0$, then $\lim \sqrt[3]{a_n} = 0$.
   (b) Prove that if $\lim a_n = a > 0$, then $\lim \sqrt[3]{a_n} = \sqrt[3]{a}$.
2. Prove that if x_n converges and y_n diverges, then $x_n + y_n$ diverges.
3. Show that if $x_n + y_n$ converges to 6 and $x_n - y_n$ converges to 3, then both x_n and y_n converge. Find $\lim x_n$ and $\lim y_n$.
4. Suppose $\dfrac{x_n}{2 + x_n}$ converges to 3. Prove that x_n converges and find it limit.
5. Let x_n be a sequence with the property that $x_n^2 - 5x_n$ converges to 14.
   (a) If x_n converges what are the only possible values of its limit?
   (b) Must x_n converge?
6. Show that the sequence $x_1 = 1$, $x_{n+1} = x_n + \dfrac{1}{x_n}$ diverges. (Hint: Suppose it did converge.)
7. Define $a_1 = 2$ and $a_{n+1} = \frac{4}{5}a_n + 5$ for $n \geq 1$. Prove that (a_n) converges, and find its limit.
8. Let $x_1 = 2$ and $x_{n+1} = \dfrac{1}{4}(1 + x_n^2)$ for $n \geq 1$. Prove that (x_n) converges, and find its limit.
9. Let $x_1 = 2$ and $x_{n+1} = \frac{x_n^2+2}{2x_n}$.
   (a) Show that $x_n \geq \sqrt{2}$ for all n.
   (b) Show that (x_n) is decreasing.
   (c) Show that (x_n) converges to $\sqrt{2}$.
10. Define $x_1 = 1$ and $x_{n+1} = x_n + \frac{1}{(n+1)^2}$ for $n \geq 1$. Prove that the sequence (x_n) converges.