Due by 9am on November 10. Please upload your solutions to Canvas as one PDF file. Do not forget to attach the honor code. You must show all your work for full credit. Each problem is worth 10 points.

(1) (a) Is $\displaystyle\int_0^{\pi/2} \cot x\, dx$ an improper integral? Explain.

(b) Find a value of $b > 0$ that makes $\displaystyle\int_0^b \frac{1}{x^2-4}\, dx$ an improper integral.

(c) Which comparison would show that $\displaystyle\int_0^\infty \frac{dx}{x+e^x}$ converges?

(d) Explain why it is not possible to draw any conclusions about the convergence of $\displaystyle\int_1^\infty \frac{e^{-x}}{x}\, dx$ by comparing with the integral $\displaystyle\int_1^\infty \frac{dx}{x}$.

(2) For which integers p does $\displaystyle\int_0^{1/2} \frac{dx}{x(\ln x)^p}$ converge?

(3) (a) Show that $\displaystyle\int_4^\infty \frac{dx}{x^2-5x+6} = \ln 2$. (b) Show that $\displaystyle\int_1^\infty \frac{1-\sin x}{x^2}\, dx$ converges.

(4) Determine whether each integral is convergent or divergent.

(a) $\displaystyle\int_0^\infty \frac{x}{(1+x^2)^2}\, dx$ (b) $\displaystyle\int_3^6 \frac{x}{\sqrt{x-3}}\, dx$ (c) $\displaystyle\int_{-\infty}^\infty xe^{-x^2}\, dx$ (d) $\displaystyle\int_{-\infty}^\infty e^{-|x|}\, dx$

(5) Use the Comparison Test to determine whether or not the integral converges.

(a) $\displaystyle\int_1^\infty \frac{1}{\sqrt{x^5+2}}\, dx$ (b) $\displaystyle\int_0^5 \frac{dx}{x^{1/3}+x^3}$ (c) $\displaystyle\int_0^\infty \frac{dx}{xe^x+x^2}$ (d) $\displaystyle\int_{-\infty}^\infty e^{-x^2}\, dx$

(6) When a capacitor of capacitance C is charged by a source of voltage V, the power expended at time t is

$$P(t) = \frac{V^2}{R}\left(e^{-t/RC} - e^{-2t/RC}\right)$$

where R is the resistance in the circuit. The total energy stored in the capacitor is

$$W = \int_0^\infty P(t)\, dt$$

Show that $W = \frac{1}{2}CV^2$?

(7) Conservation of Energy can be used to show that when a mass m oscillates at the end of a spring with spring constant k, the period of oscillation is

$$T = 4\sqrt{m}\int_0^{\sqrt{2E/k}} \frac{dx}{\sqrt{2E-kx^2}}$$

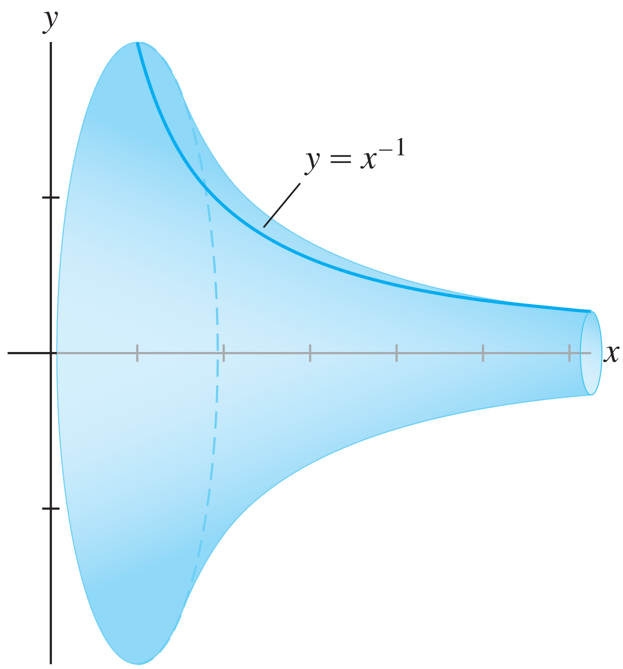
where E is the total energy of the mass. Show that this is an improper integral with value $T = 2\pi\sqrt{m/k}$.

(8) Find the volume of the solid obtained by rotating the region enclosed by the graphs about the given axis.

(a) $y = x^2,\ y = 12 - x,\ x = 0,$ about $y = -2$

(b) $x = 4 - y,\ x = 16 - y^2,$ about the y axis

(9) The solid S obtained by rotating the region below the graph of $y = x^{-1}$ about the x-axis for $1 \leq x < \infty$ is called the **Gabriel's Horn** (see the figure below).



(a) Use the Disk Method to compute the volume of S. Note that the volume is finite even though S is an infinite region.

(b) It can be shown that the surface area of S is

$$A = 2\pi \int_1^\infty x^{-1}\sqrt{1 + x^{-4}}\, dx$$

Show that A is infinite. If S were a container, you could fill its interior with a finite amount of paint, but you could not paint its surface with a finite amount of paint.

(10) Let R be the region enclosed by $y = x^2 + 2$, $y = (x - 2)^2$ and the axes $x = 0$ and $y = 0$. Compute the volume V obtained by rotating R about the x-axis. **Hint:** Express V as a sum of two integrals.