Due by 5pm on Friday, December 1. Do not forget to attach the honor code. There are five problems.

(1) Show that the dual code of a cyclic code is cyclic.

(2) Find all the possible monic generators for each of the following ideals:

(a) $I = \langle 1 + x + x^3 \rangle \subset \mathbb{F}_2[x]/(x^7 - 1)$;
(b) $I = \langle 1 + x^2 \rangle \subset \mathbb{F}_3[x]/(x^4 - 1)$.

(3) Determine whether the following polynomials are generator polynomials of cyclic codes of given lengths:

(a) $g(x) = 1 + x + x^2 + x^3 + x^4$ for a binary cyclic code of length 7;
(b) $g(x) = 2 + 2x^2 + x^3$ for a ternary cyclic code of length 8;
(c) $g(x) = 2 + 2x + x^3$ for a ternary cyclic code of length 13.

(4) Determine the smallest length for a binary cyclic code for which each of the following polynomials is the generator polynomial:

(a) $g(x) = 1 + x^4 + x^5$;
(b) $g(x) = 1 + x + x^2 + x^4 + x^6$.

(5) Let $g(x) = 1 + x^4 + x^6 + x^7 + x^8 \in \mathbb{F}_2[x]$ be the generator polynomial of a binary $[15, 7]$-cyclic code C. Write down a generator matrix and a parity-check matrix for C. Construct a generator matrix of the form $(I_7|A)$.