Due by 5pm on Friday, November 17. Do not forget to attach the honor code. There are five problems.

(1) For each of the following sets S and corresponding finite fields $\mathbb{F}_q$, find the $\mathbb{F}_q$-linear span $\langle S \rangle$ and its orthogonal complement $S^\perp$:

(a) $S = \{101, 111, 010\}$, $q = 2$
(b) $S = \{1020, 0201, 2001\}$, $q = 3$
(c) $S = \{00101, 10001, 11011\}$, $q = 2$

(2) Determine which of the following codes are linear over $\mathbb{F}_q$:

(a) $q = 2$ and $C = \{1101, 1110, 1011, 1111\}$,
(b) $q = 3$ and $C = \{0000, 1001, 0110, 2002, 1111, 0220, 1221, 2112, 2222\}$,
(c) $q = 2$ and $C = \{00000, 11110, 01111, 10001\}$.

(3) Find a generator matrix and a parity-check matrix for the linear code generated by each of the following sets, and give the parameters $[n, k, d]$ for each of these codes:

(a) $q = 2$, $S = \{1000, 0110, 0010, 0001, 1001\}$,
(b) $q = 3$, $S = \{110000, 011000, 001100, 000110, 000011\}$,
(c) $q = 2$, $S = \{10101010, 11001100, 11110000, 01100110, 00111100\}$.

(4) Find the distance of the binary linear code C with each of the following given parity-check matrices:

(a) $H = \begin{pmatrix} 0111000 \\ 1110100 \\ 1100010 \\ 1010001 \end{pmatrix}$ (b) $H = \begin{pmatrix} 1101000 \\ 1010100 \\ 0110010 \\ 1100001 \end{pmatrix}$

(5) Let C be the binary linear code with parity-check matrix

$$H = \begin{pmatrix} 110100 \\ 101010 \\ 011001 \end{pmatrix}$$

Write down a generator matrix for C and list all the codewords in C. Decode the following words:

(a) 110110 (b) 011011 (c) 101010