MATH 361 Real Analysis **Homework Assignment 4** **Spring 2024**

Due by 4pm on March 1. Do not forget to attach the honor code. Each problem is worth 10 points.

1. Prove that every subset of $\mathbb{N}$ is countable.
2. Prove that every infinite set contains a countably infinite subset.
3. Explain why the set A is countable if and only if $\text{card}(A) \leq \text{card}(\mathbb{N})$.
4. Let S be the set of all infinite sequences of zeros and ones. For example, $x = (0, 1, 0, 1, 0, 1, \ldots)$ and $y = (0, 1, 0, 1, 1, 0, 1, 1, 1, \ldots)$ are elements of S. Prove that $S \sim \mathcal{P}(\mathbb{N})$.
5. Let B be the set of all infinite subsets of $\mathbb{N}$. Prove that B is uncountable.
6. Let A be any nonempty set and let B be the set of all functions $f : A \to \{0, 1\}$. Prove that $B \sim \mathcal{P}(A)$.

   The **Schröder-Bernstein Theorem** states that if there exist injections $f : A \to B$ and $g : B \to A$ (or equivalently if there exist surjections $f : A \to B$ and $g : B \to A$) then $A \sim B$.

   Use the Schröeder-Bernstein Theorem to prove the following.

7. $\mathcal{P}(\mathbb{N}) \sim (0, 1)$.
8. $(0, 1) \times (0, 1) \sim (0, 1)$.
9. $[0, 1] \sim [0, 1)$.
10. $[0, 1] \sim (0, 1)$.