**MATH 400 Directed Reading**          **Homework Assignment 3**          **Fall 2023**

**Due by 5pm on Monday, October 23. Do not forget to attach the honor code. There is only one problem.**

**Problem 1** Consider the polynomial $x^{21} - 1$ over $\mathbb{F}_2$.

 (i) Find a complete set of representatives of cyclotomic cosets of 2 modulo 21.
 (ii) Let $\alpha$ be a root of the polynomial $x^6 + x + 1$. Show that $\alpha$ is a primitive element of $\mathbb{F}_{64}$.
(iii) Lust all cyclotomic cosets of 2 modulo 63 containing multiples of 3.
(iv) Obtain the factorization of $x^{21} - 1$ over $\mathbb{F}_2$ into monic irreducible polynomials.