Due by 4pm on February 16. Do not forget to attach the honor code.

1. (5 points each) True or False. Provide a proof or counterexample for each assertion.
   (a) If $\inf(A) < x < \sup(A)$, then $x \in A$.
   (b) If $x > \sup(A)$, then $x \notin A$.
   (c) If $x < \sup(A)$, then there exists $a \in A$ such that $x < a$.
   (d) If $a < b$ for every $a \in A$ and every $b \in B$, then $\sup(A) < \inf(B)$.
2. (15 points) Let $A = \{\frac{2n+1}{n+3} : n \in \mathbb{N}\}$. Find $\inf(A)$ and $\sup(A)$, and prove your assertions.
3. (10 points) Suppose A and B are nonempty subsets of $\mathbb{R}$ that are bounded above and $A \subseteq B$. Show that
$$\sup(A) \leq \sup(B).$$
4. Let $B = \{x \in \mathbb{R} : x(x-1)(x-2) < 10\}$.
   (a) (10 points) Prove that B is nonempty and bounded above.
   (b) (5 points) By part (a) and the completeness axiom, $\sup(B)$ exists. Estimate $\sup(B)$ to one decimal place.
5. (10 points) Prove that if A has a maximum element, then $\sup(A) = \max(A)$.
6. (10 points each)
   (a) Let A and B be nonempty subsets of $\mathbb{R}$ that are bounded above, and write
$$A + B = \{a + b : \ a \in A,\, b \in B\}.$$
   Prove that $A + B$ is bounded above and that $\sup\,(A+B) = \sup\, A + \sup\, B$.
   (b) Let A and B be nonempty sets of positive real numbers that are bounded above, and write
$$AB = \{ab : \ a \in A,\, b \in B\}.$$
   Prove that AB is bounded above and that $\sup\,(AB) = (\sup\, A)(\sup\, B)$.
7. (10 points) Let A be a nonempty subset of $\mathbb{R}$ that is bounded below. Prove that $\inf(A)$ exists.
   Hint: Let $B = \{-a : a \in A\}$ and show that B is bounded above.