Due by 9am on September 29. Please upload your solutions to Canvas as one PDF file. Do not forget to attach the honor code. Each of the first seven problems is worth 10 points. Problem 8 is worth 30 points, 5 points each. You must show all your work for full credit.

(1) Evaluate $\dfrac{d}{dx}\displaystyle\int_{\ln x}^{e^x} \sin t\, dt$.

(2) The **error function**

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, dt$$

is used in probability, statistics, and engineering.

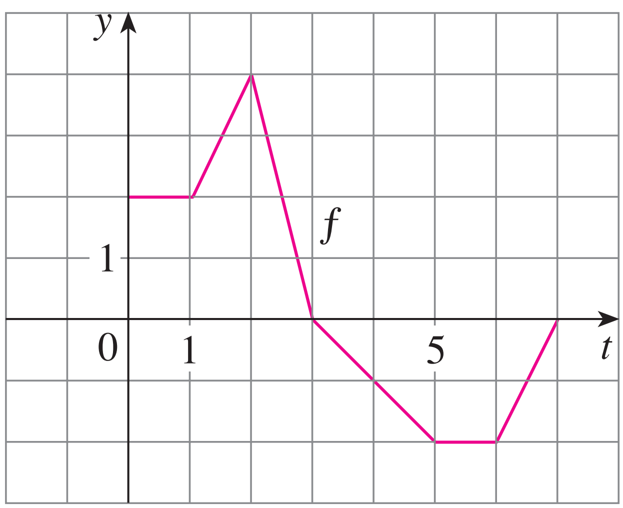
(a) Show that $\displaystyle\int_a^b e^{-t^2}\, dt = \frac{1}{2}\sqrt{\pi}\,[\operatorname{erf}(b) - \operatorname{erf}(a)]$.

(b) A differential equation is an equation that involves a function and its derivatives. Show that the function $y = e^{x^2}\operatorname{erf}(x)$ satisfies the differential equation

$$y' = 2xy + 2/\sqrt{\pi}.$$

(3) Let $g(x) = \displaystyle\int_0^x f(t)\, dt$, where f is the function whose graph is shown.

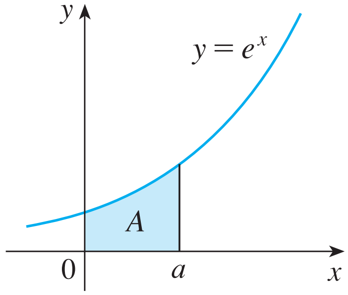
(a) Evaluate $g(0)$, $g(1)$, $g(2)$, $g(3)$, and $g(6)$.
(b) On what interval is g increasing?
(c) Where does g have a maximum value?
(d) Sketch a rough graph of g.

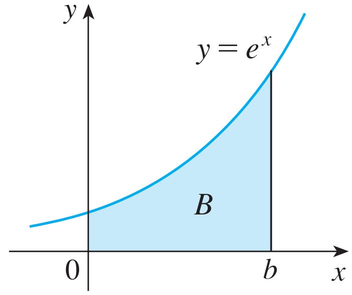


(4) Find a function f and a number a such that

$$6 + \int_a^x \frac{f(t)}{t}\, dt = 2\sqrt{x} \text{ for all } x > 0.$$

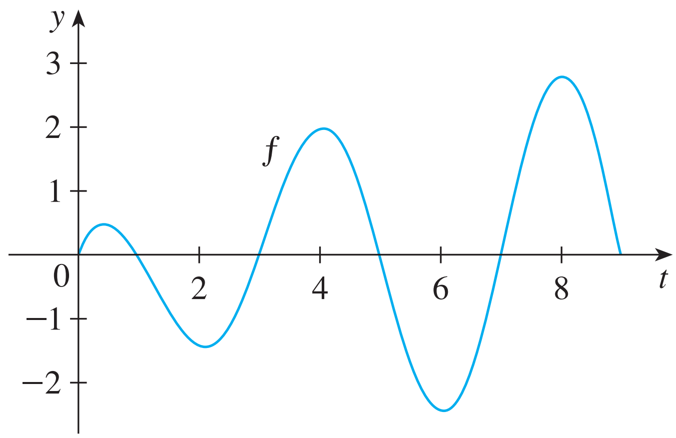
(5) The area labeled B is three times the area labeled A. Express b in terms of a.





(6) Let $g(x) = \int_0^x f(t)\,dt$, where f is the function whose graph is shown.

(a) At what values of x do the local maximum and minimum values of g occur?
(b) Where does t attain its absolute maximum value?
(c) On what intervals is g concave downward?
(d) Sketch the graph of g.



(7) The **sine integral function**

$$\mathrm{Si}(x) = \int_0^x \frac{\sin t}{t}\,dt$$

is important in electrical engineering. The integrand $f(t) = (\sin t)/t$ is not defined when $t = 0$, but we know from Calculus 1 that its limit is 1 when $t \to 0$. So we define $f(0) = 1$ and this makes f a continuous function everywhere.

(a) Draw the graph of Si.
(b) At what values of x does this function have local maximum values?
(c) Find the coordinates of the first inflection point to the right of the origin.
(d) Does this function have horizontal asymptotes?

(8) Evaluate the following integrals using FTC part II.

(i) $\displaystyle\int_0^2 10x^4 + 6x^2 - 4x\,dx$

(ii) $\displaystyle\int_1^{16} \frac{1}{\sqrt{t}}\,dt$

(iii) $\displaystyle\int_0^3 |x^2 - 4|\,dx$

(iv) $\displaystyle\int_{\pi/4}^{\pi/6} \csc\theta\cot\theta\,d\theta$

(v) $\displaystyle\int_0^5 |x^2 - 5x + 6|\,dx$

(vi) $\displaystyle\int_1^2 \frac{v^3 + 3v^6}{v^4}\,dv$