Due by 5pm on October 2. Do not forget to attach the honor code. Each problem is worth 10 points.

(1) Let $f(x) = (2+2x^2)(2+x^2+x^3)^2(-1+x^4) \in \mathbb{F}_3[x]$ and $g(x) = (1+x^2)(-2+2x^2)(2+x^2+x^3) \in \mathbb{F}_3[x]$. Determine $\gcd(f(x), g(x))$ and $\mathrm{lcm}(f(x), g(x))$.

(2) Find the order of the elements α, α^3, $\alpha+1$ and α^3+1 in $\mathbb{F}_{16}$, where α is a root of $1+x+x^4$.

(3) (a) Let α be a primitive element of $\mathbb{F}_q$. Show that α^i is also a primitive element if and only if $\gcd(i, q-1) = 1$.
(b) Determine the number of primitive elements in the following fields: $\mathbb{F}_{19}$, $\mathbb{F}_{25}$ and $\mathbb{F}_{32}$.
(c) Determine all the primitive elements of the following fields: $\mathbb{F}_7$, $\mathbb{F}_8$, $\mathbb{F}_9$

(4) Let α be a root of $1+x^3+x^4 \in \mathbb{F}_2[x]$.
(a) List all the cyclotomic cosets of 2 modulo 15.
(b) Find the minimal polynomial of $\alpha^i \in \mathbb{F}_{16}$, for all $1 \leq i \leq 14$.
(c) Find all the irreducible polynomials of degree 4 over $\mathbb{F}_2$.

Hint: Each monic irreducible polynomial of $\mathbb{F}_q[x]$ of degree m is the minimal polynomial of some element of $\mathbb{F}_{q^m}$ with respect to $\mathbb{F}_q$.

(5) (a) Find all the cyclotomic cosets of 2 modulo 31.
(b) Find the minimal polynomials of α, α^4 and α^5, where α is a root of $1+x^2+x^5 \in \mathbb{F}_2[x]$.

(6) (a) Factorize the polynomial $x^{31}-1$ over $\mathbb{F}_2$.
(b) Factorize the polynomial $x^{12}-1$ over $\mathbb{F}_5$.