Due by 9am on September 22. Please upload your solutions to Canvas as one PDF file. Do not forget to attach the honor code. Each problem is worth 10 points. You must show all your work for full credit.

(1) (a) Estimate the area under the graph of $f(x) = 1 + x^2$ from $x = -1$ to $x = 2$ using three rectangles and right end points. Then improve your estimate by using six rectangles. Sketch the curve and the approximating rectangles.
(b) Repeat part (a) using left endpoints.
(c) Repeat part (a) using midpoints.
(d) From your sketches in parts (a) – (c), which appears to be the best estimate?

(2) Evaluate

$$\lim_{N\to\infty} \frac{1}{N} \sum_{j=1}^{N} \sqrt{1 - \left(\frac{j}{N}\right)^2}$$

by interpreting it as the area of part of a familiar geometric figure.

(3) Calculate $\int_0^{10} (8 - x)\, dx$ in two ways:
(a) As the limit $\lim_{N\to\infty} R_N$
(b) Using geometry.

(4) Calculate $\int_{-1}^{4} (4x - 8)\, dx$ in two ways:
(a) As the limit $\lim_{N\to\infty} R_N$
(b) Using geometry.

(5) (a) Evaluate $\int_1^1 \sqrt{1 + x^4}\, dx$
(b) Given that $\int_0^\pi \sin^4 x\, dx = \frac{3}{8}\pi$, what is $\int_\pi^0 \sin^4 \theta\, d\theta$?

(6) Use the Midpoint Rule with the given value of n to approximate the integral. Round the answer to four decimal places.

$$\int_0^2 \frac{x}{x+1}\, dx, \quad n = 5$$

(7) Evaluate the integral using geometry

$$\int_0^2 \left(\sqrt{4x - x^2}\right) dx$$

(8) Evaluate the integral using geometry

$$\int_0^3 |\, 3x - 1\, |\; dx$$

(9) Evaluate the integral using geometry

$$\int_0^6 (|\, 12 - 4x\, | \,-\, 4)\; dx$$

(10) Evaluate the integral using properties of integrals and geometry

$$\int_{-3}^3 \left(x + \sqrt{9 - x^2}\right) dx$$