Due by 9am on September 22. Please upload your solutions to Canvas as one PDF file. Do not forget to attach the honor code. Each problem is worth 10 points.

(1) A stone, released from a state of rest, falls to earth. Compute the stone's average velocity over the time intervals $[0.8, 0.81]$, $[0.8, 0.805]$, $[0.8, 0.8001]$, $[0.8, 0.80005]$, $[0.8, 0.800001]$, and then estimate the instantaneous velocity at $t = 0.8\,\text{s}$.

Hint: Galileo's formula

(2) The formula $v = 20\sqrt{T}$ provides a good approximation to the speed of sound v in dry air (in m/s) as a function of air temperature T (in kelvins). Estimate the instantaneous rate of change of v with respect to T when $T = 273\,\text{K}$ by computing average rates of change of v over 8 intervals. What are the units of this rate?

(3) Determine the infinite limit.

(a) $\lim\limits_{x\to 3^+} \dfrac{x+1}{x-3}$

(b) $\lim\limits_{x\to 4} \dfrac{3-x}{(x-4)^2}$

(c) $\lim\limits_{x\to 2^+} \ln(x^2-4)$

(d) $\lim\limits_{x\to \frac{\pi}{2}^-} \ln(\cos x)$

(e) $\lim\limits_{x\to 3^+} \dfrac{x^2-3x}{x^2-6x+9}$

(4) (a) Find the vertical asymptotes of the function $y = \dfrac{x^2+1}{3x-2x^2}$.

(b) In the theory of relativity, the mass of a particle with velocity v is

$$m = \frac{m_0}{1 - v^2/c^2}$$

where m_0 is the mass of the particle at rest and c is the speed of light. What happens as $v \to c^-$?

(5) Let

$$\operatorname{sgn} x = \begin{cases} -1 & \text{if } x < 0, \\ 0 & \text{if } x = 0, \\ 1 & \text{if } x > 0. \end{cases}$$

(a) Sketch the graph of the function.

(b) Find each of the following limits or explain why it does not exist.

(i) $\lim\limits_{x\to 0^+} \operatorname{sgn} x$

(ii) $\lim\limits_{x\to 0^-} \operatorname{sgn} x$

(iii) $\lim\limits_{x\to 0} \operatorname{sgn} x$

(iv) $\lim\limits_{x\to 0} |\operatorname{sgn} x|$

(6) Let $g(x) = \dfrac{x^2+x-6}{|x-2|}$.

(a) Find $\lim\limits_{x\to 2^+} g(x)$

(b) Find $\lim\limits_{x\to 2^-} g(x)$

(c) Does $\lim\limits_{x\to 2} g(x)$ exist?

(d) Sketch the graph of g.

(7) Evaluate the limits.

(a) $\lim_{x\to 2} \dfrac{\sqrt{6-x}-2}{\sqrt{3-x}-1}$

(b) $\lim_{x\to 3} (2x + |x-3|)$

(8) Let

$$B(t) = \begin{cases} 4 - \frac{1}{2}t & \text{if } t < 2, \\ \sqrt{t+c} & \text{if } t \geq 2. \end{cases}$$

Find the value of c so that $\lim_{t\to 2} B(t)$ exists.

(9) Evaluate the limits.

(a) $\lim_{t\to -3} \dfrac{t^2-9}{2t^2+7t+3}$

(b) $\lim_{h\to 0} \dfrac{(2+h)^3-8}{h}$

(10) Is there a number a such that

$$\lim_{x\to -2} \frac{3x^2+ax+a+3}{x^2+x-2}$$

exists? If so, find the value of a and the value of the limit.