Due by 4pm on February 2. Do not forget to attach the honor code.

(1) (10 points) If r is rational ($r \neq 0$) and x is irrational, prove that $r + x$ and rx are irrational.

(2) (5 points each)

(a) Prove that $\sqrt{3}$ is irrational. In other words, show that there is no rational p such that $p^2 = 3$.

(b) Prove that there is no rational number whose square is 12.

(3) (10 points) Let X be a set. Then X^c denotes the complement of X. Show that $(A \cap B)^c = A^c \cup B^c$.

(4) (5 points each) Let $a, b, c, d \in \mathbb{R}$. Prove the following:

(a) $-(-a) = a$.
(b) If $a \neq 0$, then $(a^{-1})^{-1} = a$.
(c) If $a \neq 0$ and $b \neq 0$, then $(a \cdot b)^{-1} = b^{-1} \cdot a^{-1}$.
(d) If $a + b = a + c$, then $b = c$.
(e) If $a \neq 0$ and $a \cdot b = a \cdot c$ then $b = c$.
(f) $(-a) \cdot b = -(a \cdot b)$.
(g) If b and d are nonzero, then $\dfrac{a}{b} + \dfrac{c}{d} = \dfrac{a \cdot d + c \cdot b}{b \cdot d}$
(h) If $a > b > 0$ then $a^2 > b^2$.
(i) $a < 0$ if and only if $-a > 0$.
(j) If $a \neq 0$, then $a^2 > 0$.
(k) If $a > b$ and $c < 0$, then $a \cdot c < b \cdot c$.
(l) If $a > b > 0$, then $b^{-1} > a^{-1} > 0$.

(5) (10 points) Prove that if $|x - y| < \epsilon$ for every $\epsilon > 0$, then $x = y$.