Please do not write in the boxes immediately below.

Problem	1	2	3	4	5	6	7	8	9	10	11	12	Total
Points													

MATH 135 Fall 2023 Final Exam

December 15, 2023

Your name_____

The exam has 12 different printed sides of exam problems and 1 side workspace.

Duration of the Final Exam is two and a half hours. There are 12 problems, 10 points each. Only 10 problems will be graded. If you solve more than 10 problems, you must cross out the problem(s) in the box above that must not be graded. If you solve more than 10 problems and do not cross out problems, only the first ten problems will be graded. Show all your work for full credit. Books, notes etc. are prohibited. Calculators, cellphones, earphones, AirPods and cheat sheets are NOT permitted.

1. (a) Evaluate the limit $\lim_{x \to 2} \dfrac{\sqrt{4x+1} - 3}{x - 2}$

(b) Evaluate the limit using the Squeeze Theorem.

$$\lim_{t \to 0} (2^t - 1) \cos\left(\frac{1}{t}\right)$$

2. (a) Evaluate $\lim_{x \to 0^+} x^{\sin x}$

(b) Find the vertical and horizontal asymptotes, if any, of the function $f(x) = \dfrac{\sqrt{36x^2 + 7}}{9x + 4}$

3. (a) Determine where f is continuous expressing your answer in interval notation. Show all your work.

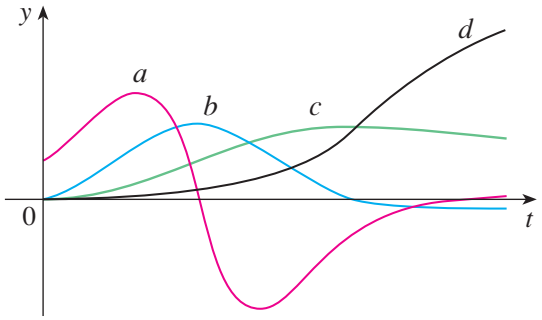
$$f(x) = \begin{cases} \dfrac{x^2 - x}{x^2 - 1} & \text{if } x \neq 1 \\ 1 & \text{if } x = 1 \end{cases}$$

(b) Let $f(x) = \dfrac{\sin x}{|x|}$. Is it continuous everywhere? Is it differentiable everywhere? Justify your answers.

4. (a) Use the limit definition to compute the derivative of the function $f(x) = \sqrt{1+2x}$ at $x = 4$.

(b) At what point on the curve $y = \sqrt{1+2x}$ is the tangent line perpendicular to the line $6x + 2y = 1$?

5. (a) The figure shows graphs of f, f', f'', and f'''. Identify each curve, and explain your choices.



(b) Find the 1000th derivative of $f(x) = xe^{-x}$

6. (a) Find an equation of the tangent line to the curve at the given point.

$$y \sin 2x = x \cos 2y, \quad (\pi/2, \pi/4)$$

(b) Use logarithmic differentiation to find dy/dx.

$$y = \frac{e^{-x} \cos^2 x}{x^2 + x + 1}$$

7. The position of a particle is given by $s(t) = \frac{4}{3}t^3 - 16t + 5$, $t \geq 0$, where t is measured in seconds and s in meters.

(a) Find the velocity at time t and after 2 seconds.

(b) When is the particle at rest?

(c) When is the particle moving forward?

(d) Find the acceleration at time t and after 2 seconds.

(e) When is the particle speeding up? When is it slowing down?

8. (a) Let $f(x) = \tan x$. Show that $f(0) = f(\pi)$ but there is no number c in $(0, \pi)$ such that $f'(c) = 0$. Why does this not contradict Rolle's Theorem?

(b) Verify that the function satisfies the hypotheses of the Mean Value Theorem on the given interval. Then find all numbers c that satisfy the conclusion of the Mean Value Theorem.

$$f(x) = 2x^2 - 3x + 1, \quad [0, 2]$$

9. (a) Sketch the graph of a function f that is continuous on $[1, 5]$ and has the given properties.

" Absolute maximum at 4, absolute minimum at 5, local maximum at 2, local minimum at 3. "

(b) Find the absolute maximum and absolute minimum values of f on the given interval. Mention any theorem used.

$$f(x) = 3x^4 - 4x^3 - 12x^2 + 1, \quad [-2, 3]$$

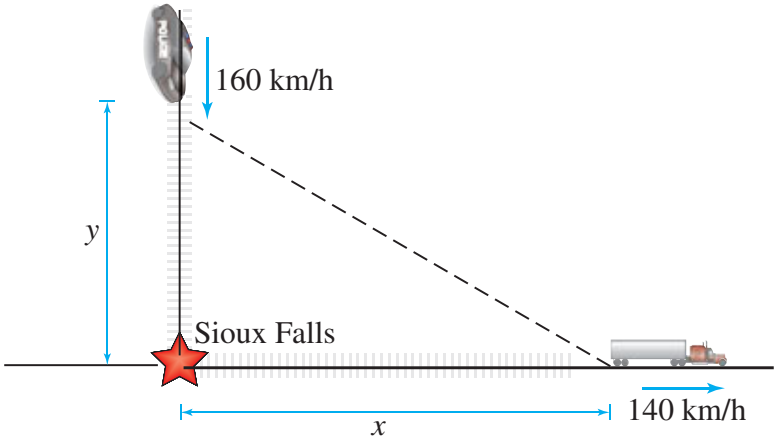
10. (a) Find the local maximum and local minimum values, if any, of $f(x) = x^3 - 3x^2 - 9x + 4$ using the First Derivative Test.

(b) Find the intervals on which f is concave up or concave down, and point(s) of inflection.

(c) Sketch the graph of f.

11. What is the minimum vertical distance between the parabolas $y = x^2 + 1$ and $y = x - x^2$?

12. A police car traveling south toward Sioux Falls at 160 km/h pursues a truck traveling east away from Sioux Falls, Iowa, at 140 km/h (figure below). At time $t = 0$, the police car is 20 km north and the truck is 30 km east of Sioux Falls. Calculate the rate at which the distance between the vehicles is changing at time $t = 0$.



Workspace