Please do not write in the boxes immediately below.

Problem	1	2	3	4	5	6	7	8	9	10	11	12	Total
Points													

MATH 135 Fall 2023 Final Exam

December 15, 2023

Your name_____

The exam has 12 different printed sides of exam problems and 1 side workspace.

Duration of the Final Exam is two and a half hours. There are 12 problems, 10 points each. Only 10 problems will be graded. If you solve more than 10 problems, you must cross out the problem(s) in the box above that must not be graded. If you solve more than 10 problems and do not cross out problems, only the first ten problems will be graded. Show all your work for full credit. Books, notes etc. are prohibited. Calculators, cellphones, earphones, AirPods and cheat sheets are NOT permitted.

1. (a) Evaluate the limit $\lim_{x \to 4} \dfrac{\sqrt{5-x}-1}{2-\sqrt{x}}$

(b) Evaluate the limit using the Squeeze Theorem.

$$\lim_{x \to 2} (x^2 - 4) \cos\left(\frac{1}{x-2}\right)$$

2. (a) Evaluate $\lim_{x \to \infty} x^{1/x^2}$

(b) Find the vertical and horizontal asymptotes, if any, of the function $f(x) = \dfrac{4x - 3}{\sqrt{25x^2 + 4x}}$

3. (a) Determine where f is continuous expressing your answer in interval notation. Show all your work.

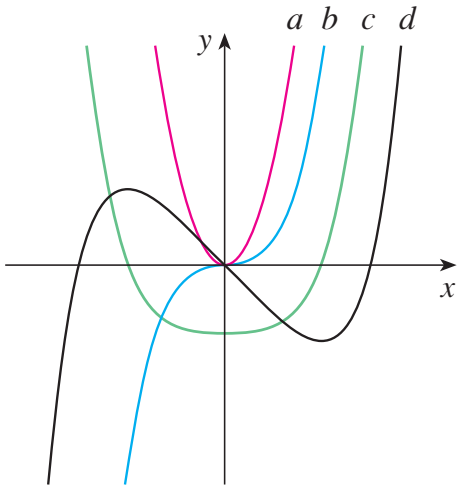
$$f(x) = \begin{cases} \dfrac{x^2 - 3x + 2}{|x - 2|} & \text{if } x \neq 2 \\ 0 & \text{if } x = 2 \end{cases}$$

(b) Let $f(x) = x + |x|$. Is it continuous everywhere? Is it differentiable everywhere? Justify your answers.

4. (a) Use the limit definition to compute the derivative of the function $f(x) = x^2 - 1$ at $x = -1$.

(b) Where does the normal line to the parabola $y = x^2 - 1$ at the point $(-1, 0)$ intersect the parabola a second time?

5. (a) The figure shows graphs of f, f', f'', and f'''. Identify each curve, and explain your choices.



(b) Find $\dfrac{d^{35}}{dx^{35}}(x \sin x)$

6. (a) Find an equation of the tangent line to the curve at the given point.

$$x^2 + \sin y = xy^2 + 1, \quad (1, 0)$$

(b) Use logarithmic differentiation to find dy/dx.

$$y = e^{x^2 - x}\,(2x + 1)^{2/3}\sqrt{x - 9}$$

7. The position of a particle is given by $s(t) = \frac{4}{3}t^3 - 16t + 5$, $t \geq 0$, where t is measured in seconds and s in meters.

(a) Find the velocity at time t and after 2 seconds.

(b) When is the particle at rest?

(c) When is the particle moving forward?

(d) Find the acceleration at time t and after 2 seconds.

(e) When is the particle speeding up? When is it slowing down?

8. (a) Show that the equation has exactly one real root.

$$2x + \cos x = 0$$

(b) Let $f(x) = (x-3)^{-2}$. Show that there is no value of c in $(1, 4)$ such that $f'(c) = \dfrac{f(4) - f(1)}{4 - 1}$. Why does this not contradict the Mean Value Theorem?

9. (a) Sketch the graph of a function f that is continuous on $[1, 5]$ and has the given properties.

" Absolute maximum at 5, absolute minimum at 2, local maximum at 3, local minima at 2 and 4. "

(b) Find the absolute maximum and absolute minimum values of f on the given interval. Mention any theorem used.

$$f(x) = x^3 - 12x^2 + 21x, \ \ [0, 2]$$

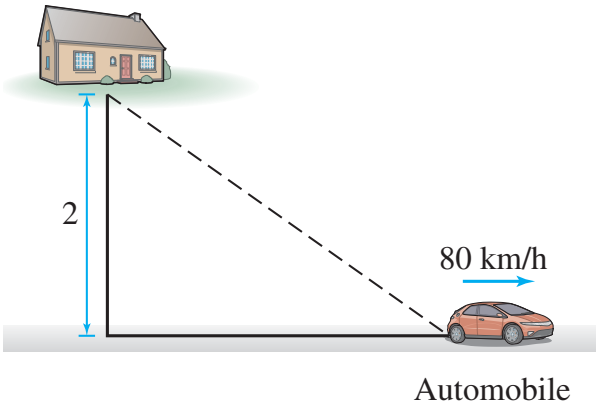
10. (a) Find the local maximum and local minimum values, if any, of $f(x) = x^3 - 12x^2 + 45x$ using the First Derivative Test.

(b) Find the intervals on which f is concave up or concave down, and point(s) of inflection.

(c) Sketch the graph of f.

11. A farmer wants to fence a rectangular garden next to his house which forms the northern boundary. The fencing for the southern boundary costs \$6 per foot, and the fencing for the east and west sides costs \$3 per foot. If he has a budget of \$120 for the project, what are the dimensions of the largest area the fence can enclose?

12. A road perpendicular to a highway leads to a farmhouse located 2 km away (figure below). An automobile travels past the farmhouse at a speed of 80 km/h. How fast is the distance between the automobile and the farmhouse increasing when the automobile is 6 km past the intersection of the highway and the road?



Workspace