Please do not write in the boxes immediately below.

Problem	1	2	3	4	5	6	7	8	9	10	11	12	Total
Points													

MATH 135 Fall 2023 Final Exam

December 15, 2023

Your name_____

The exam has 12 different printed sides of exam problems and 1 side workspace.

Duration of the Final Exam is two and a half hours. There are 12 problems, 10 points each. Only 10 problems will be graded. If you solve more than 10 problems, you must cross out the problem(s) in the box above that must not be graded. If you solve more than 10 problems and do not cross out problems, only the first ten problems will be graded. Show all your work for full credit. Books, notes etc. are prohibited. Calculators, cellphones, earphones, AirPods and cheat sheets are NOT permitted.

1. (a) Evaluate the limit $\lim_{x \to 4} \dfrac{x-4}{\sqrt{x} - \sqrt{8-x}}$

(b) Evaluate the limit using the Squeeze Theorem.

$$\lim_{x \to 1} (x-1) \sin\left(\frac{\pi}{x-1}\right)$$

2. (a) Evaluate $\lim_{x \to 0^+} \left(\frac{1}{\sin x} - \frac{1}{x} \right)$

(b) Find the vertical and horizontal asymptotes, if any, of the function $f(x) = \frac{4x - 3}{\sqrt{25x^2 + 4x}}$

3. (a) Determine where f is continuous expressing your answer in interval notation. Show all your work.

$$f(x) = \begin{cases} \dfrac{x^2 - 3x + 2}{|x - 2|} & \text{if } x \neq 2 \\ 0 & \text{if } x = 2 \end{cases}$$

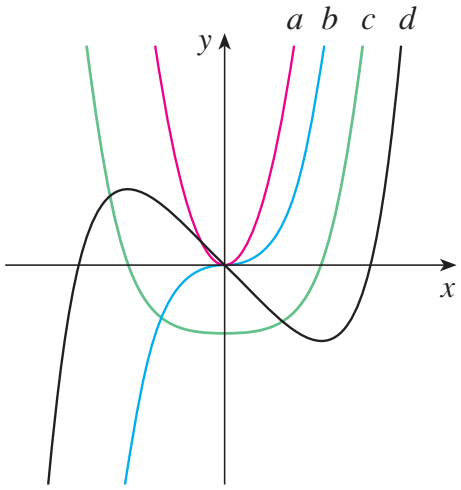
(b) How would you "remove the discontinuity" of f? In other words, how would you define $f(2)$ in order to make f continuous at 2?

$$f(x) = \frac{x^2 - x - 2}{x - 2}$$

4. (a) Use the limit definition to compute the derivative of the function $f(x) = \sqrt{x}$ at $x = 4$.

(b) Find an equation of the normal line to the curve $y = \sqrt{x}$ that is parallel to the line $2x + y = 1$.

5. (a) The figure shows graphs of f, f', f'', and f'''. Identify each curve, and explain your choices.



(b) Find $\dfrac{d^9}{dx^9}(x^8 \ln x)$

6. (a) Find an equation of the tangent line to the curve at the given point.

$$\sin(2x - y) = \frac{x^2}{y}, \quad (0, \pi)$$

(b) Use logarithmic differentiation to find dy/dx.

$$y = \sqrt{x}\, e^{x^2 - x}\, (x + 1)^{2/3}$$

7. The position of a particle is given by $s(t) = \frac{4}{3}t^3 - 16t + 5$, $t \geq 0$, where t is measured in seconds and s in meters.

(a) Find the velocity at time t and after 2 seconds.

(b) When is the particle at rest?

(c) When is the particle moving forward?

(d) Find the acceleration at time t and after 2 seconds.

(e) When is the particle speeding up? When is it slowing down?

8. (a) Two runners start a race at the same time and finish in a tie. Prove that at some time during the race they have the same speed. Mention any theorem used.

Hint: Consider $f(t) = g(t) - h(t)$, where g and h are the position functions of the two runners.

(b) At 2:00 PM a car's speedometer reads 30 mi/h. At 2:10 PM it reads 50 mi/h. Show that at some time between 2:00 and 2:10 the acceleration is exactly 120 mi/h^2. Mention any theorem used.

9. (a) Sketch the graph of a function f that is continuous on $[1, 5]$ and has the given properties.

" Absolute maximum at 2, absolute minimum at 5, 4 is a critical number but there is no local maximum or minimum there."

(b) Find the absolute maximum and absolute minimum values of f on the given interval. Mention any theorem used.

$$f(x) = 2x^3 - 3x^2 - 12x + 1, \ \ [-2, 3]$$

10. (a) Find the local maximum and local minimum values, if any, of $f(x) = 3x^4 - 8x^3 + 6x^2$ using the First Derivative Test.

(b) Find the intervals on which f is concave up or concave down, and point(s) of inflection.

(c) Sketch the graph of f.

11. We need to enclose a rectangular field with a fence. We have 500 feet of fencing material and a building is on one side of the field and so won't need any fencing. Determine the dimensions of the field that will enclose the largest area.

12. A conical tank has height 3 m and radius 2 m at the top. Water flows in at a rate of $2\,\mathrm{m}^3/\mathrm{min}$. How fast is the water level rising when it is 2 m?

Workspace