Please do not write in the boxes immediately below.

Problem	1	2	3	4	5	6	7	8	9	10	11	12	Total
Points													

MATH 136 Fall 2023 Final Exam

December 15, 2023

Your name____

The exam has 12 different printed sides of exam problems and 1 side workspace.

Duration of the Final Exam is two and a half hours. There are 12 problems, 10 points each. Only 10 problems will be graded. If you solve more than 10 problems, you must cross out the problem(s) in the box above that must not be graded. If you solve more than 10 problems and do not cross out problems, only the first ten problems will be graded. Show all your work for full credit. Books, notes etc. are prohibited. Calculators, cellphones, earphones, AirPods and cheat sheets are NOT permitted.

1. Let A be the area under the graph of $f(x) = 2x^2 - 3x + 1$ over $[1, 3]$. Compute A as the limit $\lim_{n\to\infty} R_n$.

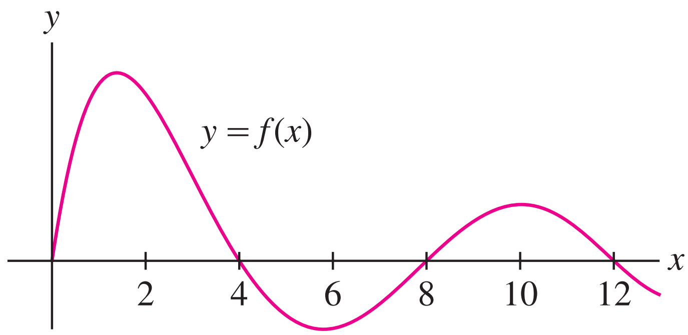
   Hint: $\displaystyle\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}, \quad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

2. (a) Evaluate the following integral using geometry. You are not allowed to use properties of definite integrals.

$$\int_{-3}^{0} \left(1 + \sqrt{9 - x^2}\right)\, dx$$

(b) Estimate $\displaystyle\int_0^1 \frac{dx}{\sqrt{5x^3 + 4}}$.

3. Let $A(x) = \int_0^x f(t)\,dt$, where f is the function whose graph is shown below. Determine:



(a) The intervals on which $A(x)$ is increasing and decreasing

(b) The values x where $A(x)$ has a local min or max

(c) The intervals where $A(x)$ is concave up or concave down

(d) The inflection points of $A(x)$

4. A particle moves in a straight line with the given velocity (in m/s).

$$v(t) = 36 - 24t + 3t^2$$

(a) Find the displacement over the time interval $[0, 10]$.

(b) **Set up an integral** to find the total distance traveled by the particle over the time interval $[0, 10]$. Your answer must not involve an absolute value function.

5. (a) **Set up an integral** to find the area between the circles $x^2 + y^2 = 2$ and $x^2 + (y - 1)^2 = 1$.

(b) **Set up an integral** to find the volume of the solid obtained by rotating the region enclosed by the graphs of $y = 2\sqrt{x}$ and $y = x$ about the horizontal axis $y = 4$.

Hint: $2\sqrt{x} > x$ for $0 < x < 4$.

6. (a) First make a substitution and then use integration by parts to evaluate the integral $\int_0^{\pi} e^{\cos t} \sin 2t\, dt$

Hint: $\sin 2t = 2\sin t \cos t$

(b) Evaluate the integral $\int \frac{dx}{x^2\sqrt{4-x^2}}$ **Hint:** $\sin^2\theta + \cos^2\theta = 1$

7. Find the exact length of the curve.

$$y = \ln(1 - x^2), \quad 0 \leq x \leq \frac{1}{2}$$

8. (a) Determine whether the integral is convergent or divergent.

$$\int_0^3 \frac{dx}{\sqrt{9-x^2}}$$

(b) Use the Comparison Test for Improper Integrals to determine whether the integral is convergent or divergent.

$$\int_1^\infty \frac{1}{x^4+e^x}\,dx$$

9. Consider the sequence $a_n = \dfrac{1-n}{2+n}$.

(a) Determine whether the sequence is increasing, decreasing, or not monotonic.

(b) Is it bounded above?

(c) Is it bounded below?

(d) Is it convergent?

10. (a) Determine whether the series is convergent or divergent by expressing the nth partial sum S_n as a telescoping sum. If it is convergent, find its sum.

$$\sum_{n=1}^{\infty} \frac{3}{n(n+3)}$$

(b) Determine whether the series is convergent or divergent. Mention any Convergence Test used.

$$\sum_{n=1}^{\infty} \ln\left(\frac{n^2+1}{2n^2+1}\right)$$

11. (a) Use the Limit Comparison Test to determine whether the series is convergent or divergent.

$$\sum_{n=1}^{\infty} \frac{(2n-1)(n^2-1)}{(n+1)(n^2+4)^2}$$

(b) Use the Integral Test to determine whether the infinite series is convergent or divergent.

$$\sum_{n=1}^{\infty} \frac{1}{n^2+4}$$

12. Find the radius of convergence and interval of convergence of the following series.

$$\sum_{n=0}^{\infty} \frac{(-3)^n\, x^n}{\sqrt{n+1}}$$

Workspace