Please do not write in the boxes immediately below.

problem	1	2	3	4	5	6	7	total
points								

MATH 361 Spring 2024 Exam 3

April 25, 2024

Your name____

The exam has 7 different printed sides of exam problems and 1 side workspace.

Duration of the Midterm Exam is 90 minutes. There are 7 problems. The first problem is mandatory, and it is worth 10 points. Each of the Problems 2 – 7 is worth 8 points. From Problems 2 – 7, only 5 problems will be graded. If you solve all Problems 2 – 7, you must cross out the problem in the box above that must not be graded. If you solve all Problems 2 – 7 and do not cross out a problem, only the first five problems from 2 – 7 will be graded. Show all your work for full credit. Books, notes etc. are prohibited. Calculators, cellphones, earphones, AirPods and cheat sheets are NOT permitted.

1. (a) Define the following:

    (i) Open set.

    (ii) Limit point of a set A.

   (b) Identify all the limit points of the set $A = (1, 3)$.

   (c) Provide a sequence contained in the set that converges to each limit point in part (b) where each a_n is not equal to the limit point.

   (d) Decide whether the following set is open, closed, or neither. If the set is not open, find a point in the set for which there is no ϵ-neighborhood contained in the set. If the set is not closed, find a limit point that is not contained in the set.

$$\{1 + 1/2 + 1/3 + \cdots + 1/n \; : \; n \in \mathbb{N}\}$$

2. (a) State the Cauchy Condensation Test.

(b) Prove that a p-series $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^p}$ converges if and only if $p > 1$.

(c) Determine whether the series $\displaystyle\sum_{k=2}^{\infty} \frac{1}{k(\ln k)^2}$ converges or diverges.

3. (a) State the Direct Comparison Test.

(b) Give a proof for the Direct Comparison Test using the Monotone Convergence Theorem.

(c) Determine whether the series $\sum_{n=1}^{\infty} \frac{2n^2+n+1}{n^4+n+1}$ is convergent or divergent.

4. (a) Define the absolute convergence of a series $\sum_{k=1}^{\infty} a_k$.

(b) Prove the following: If a series converges absolutely, then it converges. Mention any result used.

(c) Define the conditional convergence of a series $\sum_{k=1}^{\infty} a_k$.

(d) Give an example of a series that converges conditionally.

(e) Give an example of a series that is neither absolutely convergent nor conditionally convergent.

5. Find the exact value of the sum of each series.

(a) $\displaystyle\sum_{k=1}^{\infty} \frac{5^{2k}}{100^{k-1}}$

(b) $\displaystyle\sum_{k=1}^{\infty} a_k$, where $a_1 + a_2 + \cdots + a_n = \dfrac{3n+1}{2n+5}$ for all n.

6. Determine whether the following series converges or diverges.

(a) $\displaystyle\sum_{k=1}^{\infty} \frac{(k!)^3}{(3k)!}$

(b) $\displaystyle\sum_{k=0}^{\infty} (\arctan k)^k$

(c) $\displaystyle\sum_{k=2}^{\infty} \frac{\cos k\pi}{(\ln k)^2}$

7. (a) Complete the following definition. $\lim_{x \to c} f(x) = L$ if for every $\epsilon > 0$,

(b) Use the definition to prove $\lim_{x \to 3} x^2 + 4x = 21$.

WORKSHEET