

1. Find a formula for the described function and state its domain.

(i) A rectangle has perimeter 20m. Express the area of the rectangle as a function of the length of one of its sides.

(ii) A rectangle has area 16m^2 . Express the perimeter of the rectangle as a function of the length of one of its sides.

(iii) Express the area of an equilateral triangle as a function of the length of a side.

(iv) A closed rectangular box with volume 8ft^3 has length twice the width. Express the height of the box as a function of the width.

(v) An open rectangular box with volume 2m^3 has a square base. Express the surface area of the box as a function of the length of a side of the base.

Piecewise Defined Functions

2. Let

$$f(x) = \begin{cases} x^2 - 1 & \text{if } x \leq 0 \\ x - 1 & \text{if } 1 \leq x \leq 4 \\ 5 & \text{if } x > 4 \end{cases}$$

(a) Compute $f(0)$, $f(2)$, $f(5)$ and $f(-1)$.

(b) Graph the function.

3. Let

$$f(x) = \begin{cases} (x - 3)^2 + 2 & \text{if } x \neq 3 \\ 4 & \text{if } x = 3 \end{cases}$$

(a) Compute $f(-1)$, $f(3)$ and $f(5)$.

(b) Graph the function.

4. Let

$$f(x) = \begin{cases} 2^x & \text{if } x \leq 1 \\ 3 - x & \text{if } 1 < x \leq 4 \\ \sqrt{x} & \text{if } x > 4 \end{cases}$$

(a) Compute $f(0)$, $f(2)$, $f(5)$ and $f(-1)$.

(b) Graph the function.

5. A cell phone plan has a basic charge of \$35 a month. The plan includes 400 free minutes and charges 10 cents for each additional minute of usage. Write the monthly cost C as a function of the number x of minutes used and graph C as a function of x for $0 < x < 600$.

6. An electricity company charges its customers a base rate of \$10 a month, plus 6 cents per kilowatt-hour (kWh) for the first 1200 kWh and 7 cents per kWh for all usage over 1200 kWh. Express the monthly cost E as a function of the amount x of electricity used. Then graph the function E for $0 < x < 2000$.

Absolute Value Functions

7. Sketch the graph of the following functions.

(a) $f(x) = |x - 3|$

(b) $g(x) = |2x + 1|$

(c) $f(x) = x + |x|$

Transformations of Functions

Vertical and Horizontal Shifts Suppose $c > 0$. To obtain the graph of

- $y = f(x) + c$, shift the graph of $y = f(x)$ a distance c units upward.
- $y = f(x) - c$, shift the graph of $y = f(x)$ a distance c units downward.
- $y = f(x - c)$, shift the graph of $y = f(x)$ a distance c units to the right.
- $y = f(x + c)$, shift the graph of $y = f(x)$ a distance c units to the left.

Vertical and Horizontal Stretching and Reflecting Suppose $c > 1$. To obtain the graph of

- $y = cf(x)$, stretch the graph of $y = f(x)$ vertically by a factor of c .
- $y = (1/c)f(x)$, shrink the graph of $y = f(x)$ vertically by a factor of c .
- $y = f(cx)$, shrink the graph of $y = f(x)$ horizontally by a factor of c .
- $y = f(x/c)$, stretch the graph of $y = f(x)$ horizontally by a factor of c .
- $y = -f(x)$, reflect the graph of $y = f(x)$ about the x -axis.
- $y = f(-x)$, reflect the graph of $y = f(x)$ about the y -axis.

1. Given the graph of $y = \sqrt{x}$, use transformations to graph

- (i) $y = \sqrt{x} - 2$,
- (ii) $y = \sqrt{x - 2}$,
- (iii) $y = -\sqrt{x}$,
- (iv) $y = 2\sqrt{x}$, and
- (v) $y = \sqrt{-x}$.

2. Sketch the graph of the function $f(x) = x^2 + 6x + 10$.