Definiton A **critical number (point)** of a function f is a number c in the domain of f such that either $f'(c) = 0$ or $f'(c)$ does not exist.

1. Find the critical points (numbers) of the function.
   (i) $f(x) = 2x^3 - 3x^2 - 36x$
   (ii) $f(t) = t^4 + t^3 + t^2 + 1$
   (iii) $f(t) = |3t - 4|$
   (iv) $f(x) = x^2 e^{-3x}$
   (v) $g(x) = \sqrt[3]{4 - x^2}$
   (vi) $F(x) = x^{4/5}(x - 4)^2$
   (vii) $f(x) = x\sqrt{x - x^2}$
   (viii) $g(y) = \dfrac{y - 1}{y^2 - y + 1}$

Definiton Let $f(x)$ be a function on an interval I and let $a \in I$.

• f attains an absolute maximum value of $f(a)$ at a if and only if $f(x) \leq f(a)$ for all $x \in I$.
• f attains an absolute minimum value of $f(a)$ at a if and only if $f(x) \geq f(a)$ for all $x \in I$.
• If f attains an absolute maximum or minimum value at a, then we say that f **attains an absolute extreme value of** $f(a)$ **at** a**.**

Note Frequently, we are interested in when the value of the function at a point is the biggest or smallest among all nearby points; this is the question of local maximum and minimum values. We usually shorten *maximum and minimum values* to *maxima* and *minima*, and lump both terms together by referring to *extrema*.

Definiton We say that $f(x)$ has a

• **Local minimum** at $x = c$ if $f(c)$ is the minimum value of $f(x)$ on some open interval (in the domain of f) containing c.
• **Local maximum** at $x = c$ if $f(c)$ is the maximum value of $f(x)$ on some open interval (in the domain of f) containing c.

Note Absolute/local extreme values are sometimes referred to as global/relative extreme values.

The following theorem says that the functions continuous on a closed interval always attain extreme values.

Extreme Value Theorem If f is continuous on a closed interval $[a, b]$, then f attains an absolute maximum value $f(c)$ and an absolute minimum value $f(d)$ at some numbers c and d in $[a, b]$.

Note An extreme value can be taken on more than once.

Note To find the absolute maximum and minimum values of a continuous function f on a closed interval $[a, b]$:

(1) Find the values of f at the critical points of f in (a, b).
(2) Find the values of f at the endpoints of the interval.
(3) The largest of the values from Steps 1 and 2 is the absolute maximum value; the smallest of these values is the absolute minimum value.

2. Explain the difference between an absolute (global) minimum and a local minimum.

3. Find the absolute maximum and absolute minimum values of f on the given interval.

   (i) $f(x) = 2x^3 - 3x^2 - 12x + 1,\ [-2, 3]$

   (ii) $f(t) = 2\cos t + \sin(2t),\ [0, \pi/2]$

   (iii) $f(x) = xe^{x/2},\ [-3, 1]$

4. (a) Find the absolute extrema of $f(x) = 2\cos x - |x|$ on the interval $[-\pi, \pi]$.

   (b) If the graph of the function $f(x) = x^2 + ax + b$ has an absolute minimum at (2,4) on $[-4, 4]$, determine the value of $f(-1)$.

Applications

5. After an antibiotic tablet is taken, the concentration of the antibiotic in the bloodstream is modeled by the function
$$C(t) = 8(e^{-0.4t} - e^{-0.6t})$$
where the time t is measured in hours and C is measured in μg/mL. What is the maximum concentration of the antibiotic during the first 12 hours?

6. The water level, measured in feet above mean sea level, of Lake Lanier in Georgia, USA, during 2012 can be modeled by the function
$$L(t) = 0.01441t^3 - 0.4177t^2 + 2.703t + 1060.1$$
where t is measured in months since January 1, 2012. Estimate when the water level was highest during 2012.