1. Find the derivative of the function using the definition of derivative. State the domain of the function and the domain of its derivative.
$$G(t) = \frac{1-2t}{3+t}$$

2. Calculate y'.

(i) $y = (x^2 + x^3)^4$

(ii) $y = \dfrac{x^2 - x + 2}{\sqrt{x}}$

(iii) $y = x^2 \sin(\pi x)$

(iv) $y = \ln(x \ln x)$

(v) $y = \dfrac{e^{1/x}}{x^2}$

(vi) $y = \sqrt{\arctan x}$

(vii) $y = \tan\left(\dfrac{t}{1+t^2}\right)$

(vii) $y = 3^{x \ln x}$

(ix) $y = e^{x \sec x}$

(x) $y = \log_5(1 + 2x)$

(xi) $y = \ln(\sin x) - \frac{1}{2}\sin^2 x$

(xii) $y = x \tan^{-1}(4x)$

(xiii) $y = \cot(3x^2 + 5)$

(xiv) $y = \dfrac{\sqrt{x+1}\,(2-x)^5}{(x+3)^7}$

(xv) $y = \sin\left(\tan\sqrt{1+x^3}\right)$

(xvi) $y = (1 - x^{-1})^{-1}$

(xvii) $y = \tan^2(\sin\theta)$

(xvii) $y = \dfrac{1}{\sqrt{x}} - \dfrac{1}{\sqrt[5]{x^3}}$

(xix) $y = \dfrac{\tan x}{1 + \cos x}$

(xx) $y = x \cos^{-1} x$

(xxi) $y = \ln \sec x$

(xxii) $y = \cot(\csc x)$

(xxiii) $y = e^{x \tan x}$

(xxiv) $y = \dfrac{x}{2 - \tan x}$

(xxv) $y = \left(x + \dfrac{1}{x}\right)^5$

(xxvi) $f(t) = 2^{t^3}$

(xxvii) $s(t) = \sqrt{\dfrac{1+\sin t}{1+\cos t}}$

(xxvii) $y = \cot^2(\sin\theta)$

(xxix) $y = \cos\left(\dfrac{1-e^{2x}}{1+e^{2x}}\right)$

(xxx) $y = \sqrt{x + \sqrt{x + \sqrt{x}}}$

(xxxi) $y = 2^{3^{4^x}}$

(xxxii) $f(t) = \sin^2(e^{\sin^2 t})$

(xxxiii) $y = \sqrt{1 + xe^{-2x}}$

(xxxiv) $y = \cos\sqrt{\sin(\tan \pi x)}$

(xxxv) $y = [x + (x + \sin^2 x)^3]^4$

(xxxvi) $f(t) = \tan(\sec(\cos t))$

3. Find equations of the tangent line and normal line to the given curve at the specified point.

(a) $y = 2xe^x$, $(0, 0)$

(b) $y = \dfrac{2x}{x^2+1}$, $(1, 1)$

4. If $g(x) = x/e^x$, find $g^{(n)}(x)$.

5. How many tangent lines to the curve $y = x/(x+1)$ pass through the point $(1, 2)$? At which points do these tangent lines touch the curve?

6. Find equations of the tangent lines to the curve
$$y = \frac{x-1}{x+1}$$
that are parallel to the line $x - 2y = 2$.

7. Find $R'(0)$, where
$$R(x) = \frac{x - 3x^3 + 5x^5}{1 + 3x^3 + 6x^6 + 9x^9}$$
Hint: Instead of finding $R'(x)$ first, let $f(x)$ be the numerator and $g(x)$ the denominator of $R(x)$ and compute $R'(0)$ from $f(0)$, $f'(0)$, $g(0)$, and $g'(0)$.

8. Find $Q'(0)$, where
$$Q(x) = \frac{1 + x + x^2 + xe^x}{1 - x + x^2 - xe^x}$$

9. Prove that $\dfrac{d}{dx}(\tan x) = \sec^2 x$

10. Prove that $\dfrac{d}{dx}(\cot x) = -\csc^2 x$

11. Prove that $\dfrac{d}{dx}(\sec x) = \sec x \tan x$

12. Prove that $\dfrac{d}{dx}(\csc x) = -\csc x \cot x$

13. For what values of x does the graph of f have a horizontal tangent?

(a) $f(x) = x + 2\sin x$

(b) $f(x) = e^x \cos x$

14. Find an equation of the tangent line to the curve at the given point.

(a) $y = \sin x + \cos x,\ (0, 1)$

(b) $y = e^x \cos x,\ (0, 1)$

(c) $y = x + \tan x,\ (\pi, \pi)$

(d) $y = \sin(\sin x),\ (\pi, 0)$

(e) $y = xe^{-x^2},\ (0, 1)$

15. Find y' and y''.

(a) $y = \cos(\sin(3\theta))$

(b) $y = \sqrt{1 - \sec t}$

(c) $y = e^{e^x}$

(d) $y = \dfrac{1}{(1 + \tan x)^2}$

16. Find the given derivative by finding the first few derivatives and observing the pattern that occurs.

(a) $\dfrac{d^{99}}{dx^{99}}(\sin x)$

(b) $\dfrac{d^{35}}{dx^{35}}(x \sin x)$

17. Find the derivative of the function. Simplify where possible.

(a) $y = (\tan^{-1} x)^2$

(b) $y = \sin^{-1}(2x + 1)$

(c) $g(x) = \cos^{-1}(\sqrt{x})$

(d) $y = \tan^{-1}(x - \sqrt{1 + x^2})$

(e) $y = \arctan \sqrt{\dfrac{1 - x}{1 + x}}$

(f) $y = \cos^{-1}(\sin^{-1} t)$

18. Differentiate the function. In part (h), a is a constant.

(a) $f(x) = x \ln x - x$

(b) $f(x) = \sin(\ln x)$

(c) $f(x) = \ln(\sin^2 x)$

(d) $g(x) = \ln(xe^{-2x})$

(e) $f(x) = \log_{10} \sqrt{x}$

(f) $h(x) = \ln(x + \sqrt{x^2 - 1})$

(g) $G(y) = \ln \dfrac{(2y + 1)^5}{\sqrt{y^2 + 1}}$

(h) $H(z) = \ln \sqrt{\dfrac{a^2 - z^2}{a^2 + z^2}}$.

(i) $y = \log_2(x \log_5 x)$

(j) $y = \ln(\csc x - \cot x)$

19. Find an equation of the tangent line to the curve at the given point.

(a) $y = \ln(x^2 - 3x + 1),\ (3, 0)$

(b) $y = x^2 \ln x,\ (1, 0)$

(c) $y = (\ln x)/x,\ (1, 0)$

(d) $y = (\ln x)/x,\ (e, 1/e)$

20. Find y' and y''.

(a) $y = \sqrt{x} \ln x$

(b) $y = \dfrac{\ln x}{1 + \ln x}$

(c) $y = \ln|\sec x|$

(d) $y = \ln(1 + \ln x)$

21. Use logarithmic differentiation to find the derivative of the function.

(a) $y = (x^2 + 2)^2(x^4 + 4)^4$

(b) $y = \dfrac{e^{-x}\cos^2 x}{x^2 + x + 1}$

(c) $y = \ln \sqrt{\dfrac{x - 1}{x^4 + 1}}$.

(d) $y = \sqrt{x} e^{x^2 - x}(x + 1)^{2/3}$

(e) $y = x^{\sin x}$

(f) $y = x^x$

(g) $y = (\ln x)^{\cos x}$

(h) $y = (\cos x)^x$

(i) $y = \tan x)^{1/x}$

(j) $y = \sqrt{x}^{\,x}$

22. Find a formula for $f^{(n)}(x)$ if $f(x) = \ln(x - 1)$.

23. Find $\dfrac{d^9}{dx^9}(x^8 \ln x)$

24. Use the definition of the derivative to prove that

$$\lim_{x \to 0} \frac{\ln(1 + x)}{x} = 1$$

25. Show that

$$\lim_{n \to \infty} \left(1 + \frac{x}{n}\right)^n = e^x \text{ for any } x > 0.$$