MATH 133 - 01 Calculus 1 with Fundamentals   Midterm Exam 2   November 16, 2022

Your Name:

Duration of the exam is 90 minutes. There are six problems, worth 50 points. Show all your work for full credit. Books, notes etc. are prohibited. Calculators are NOT permitted.

1.   (a) (4 pts) Evaluate the limit.

$$\lim_{x \to -2} \frac{x^2 + 3x + 2}{x + 2}$$

         (b) (5 pts) Evaluate the limit. The following limit gives the derivative of a function f at a point a, $f'(a)$. Find f and a.

$$\lim_{x \to 8} \frac{\sqrt{x - 4} - 2}{x - 8}$$

2.   (a) (5 pts) Compute the limit

$$\lim_{x \to 0} \frac{\sin 3x \sin 2x}{x \sin 5x}$$

(b) (5 pts) Compute the limit using the Squeeze Theorem.

$$\lim_{t \to 0} (2^t - 1) \cos\left(\frac{1}{t}\right)$$

3. Evaluate each of the following limits. Identify any vertical asymptotes of the function.

(a) (3 pts) $\lim\limits_{x \to 1} \dfrac{2 - x}{(x - 1)^2}$

(b) (3 pts) $\lim\limits_{x \to 3^-} \dfrac{\sqrt{x}}{(x - 3)^5}$

4. (6 pts) Determine all values of the constants a and b so that the following function is continuous for all values of x.

$$f(x) = \begin{cases} \dfrac{1}{x} & \text{for } x < -1 \\ ax + b & \text{for } -1 \le x \le \frac{1}{2} \\ \dfrac{1}{x} & \text{for } x > \frac{1}{2} \end{cases}$$

5.   (a) (5 pts) Let $f(x) = 4 - x^2$. Use the limit definition to compute $f'(-1)$.

(b) (3 pts) Find an equation of the tangent line to $f(x)$ in part (a) at the point $(-1, 3)$.

(c) (3 pts) Let $f(x) = x^3$ and $g(x) = x^2 + 5x$. You are given that $f'(x) = 3x^2$ and $g'(x) = 2x + 5$. Find the x values where the graphs of f and g have parallel tangent lines.

6.   (a) (3 pts) Determine the values of x at which the function in the following figure is: (i) discontinuous, and (ii) nondifferentiable.
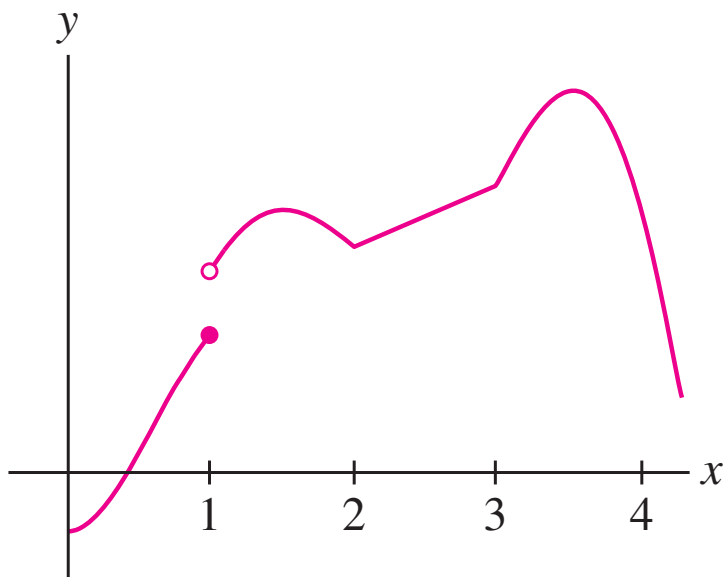
(b) (5 pts) Use the Intermediate Value Theorem to show that the equation

$$x^7 + 3x = 10$$

has a solution in the interval $[1, 2]$.