

Your Name:

Duration of the exam is 90 minutes. There are six problems, worth 50 points. Show all your work for full credit. Books, notes etc. are prohibited. Calculators are NOT permitted.

1. (i) (3 pts) Determine the domain of the function.

$$f(x) = \frac{\sqrt{x}}{x^2 - 9}$$

Hint: $a^2 - b^2 = (a - b)(a + b)$

- (ii) (3 pts) Find the range of the function

$$f(x) = \sqrt{x + 1}$$

2. Let

$$f(x) = 2^x + 1, \quad g(x) = x - 1$$

(i) (3 pts) Compute the composite function $f \circ g$ and discuss its domain.

(ii) (4 pts) Graph the function $f \circ g$.

(iii) (3 pts) Is the function $f \circ g$ one-to-one? Justify your answer.

3. (i) (3 pts) Is $f(x) = \left(\frac{1}{3}\right)^x$ increasing or decreasing? Explain clearly.

- (ii) (7 pts) Let

$$f(x) = \begin{cases} x + 1 & \text{if } x \leq -1 \\ x^2 & \text{if } x > -1 \end{cases}$$

Evaluate $f(-3)$, $f(0)$, and $f(2)$ for the piecewise-defined function. Then sketch the graph of the function.

4. (i) (3 pts) Determine whether the function is even, odd, or neither.

$$f(x) = 2^{-x^2}$$

- (ii) (4 pts) Identify each of the following functions as polynomial, rational, algebraic, or transcendental.

(a) $f(x) = 4x^3 + 9x^2 - 8$

(c) $f(x) = 2^x$

(b) $f(x) = \sin(x^2)$

(d) $f(x) = \frac{x}{\sqrt{x} + 1}$

5. (i) (4 pts) Assume that $\sin \theta = \frac{4}{5}$, where $\pi/2 < \theta < \pi$. Find $\tan \theta$ and $\sin 2\theta$.

Hint: $\sin 2\theta = 2 \sin \theta \cos \theta$

- (ii) (6 pts) You are given a point on the unit circle centered at the origin. Find the x coordinate and the values of the six trigonometric functions.

$$P\left(x, \frac{\sqrt{7}}{3}\right), \quad x < 0$$

6. (i) (4 pts) Solve for $0 \leq \theta < 2\pi$.

$$\cos \theta = -\frac{1}{2}$$

- (ii) (3 pts) Verify that the following equation is an identity.

$$\cot \theta + \tan \theta = \sec \theta \csc \theta$$

Hint: $\sin^2 \theta + \cos^2 \theta = 1$