Please do not write in the boxes immediately below.

problem	1	2	3	4	5	6	7	8	9	10	11	12	total
points													

MATH 133 Fall 2022 Final Exam

December 15, 2022

Your name____

The exam has 12 different printed sides of exam problems and 1 side workspace.

Duration of the Final Exam is two and a half hours. There are 12 problems, 10 points each. Only 10 problems will be graded. If you solve more than 10 problems, you must cross out the problem(s) in the box above that must not be graded. If you solve more than 10 problems and do not cross out problems, only the first ten problems will be graded. Show all your work for full credit. Books, notes etc. are prohibited. Calculators, cellphones, earphones, AirPods and cheat sheets are NOT permitted.

1) Let $f(x) = 2^x, g(x) = x^2$.

(i) (4 points) Compute the composite functions $f \circ g$ and $g \circ f$.

(ii) (3 points) Determine the domain of $f \circ g$.

(iii) (3 points) Find the range of $f \circ g$.

2)

(i) (1 point each) Identify each of the following functions as polynomial, rational, algebraic, or transcendental.

(a) $f(x) = x^2 - 3x + 1$

(d) $f(x) = x^2 + 3x^{-1}$

(b) $f(x) = 5^x$

(e) $f(x) = \sqrt{1 - x^2}$

(c) $f(x) = \dfrac{2x^3 + 3x}{9 - 7x^2}$

(f) $f(x) = \sin(3^x)$

(ii) (2 points each) Determine whether the function is even, odd, or neither.

(a) $f(t) = \dfrac{1}{t^4 + t + 1} - \dfrac{1}{t^4 - t + 1}$

(b) $H(\theta) = \sin(\theta^2)$

3)

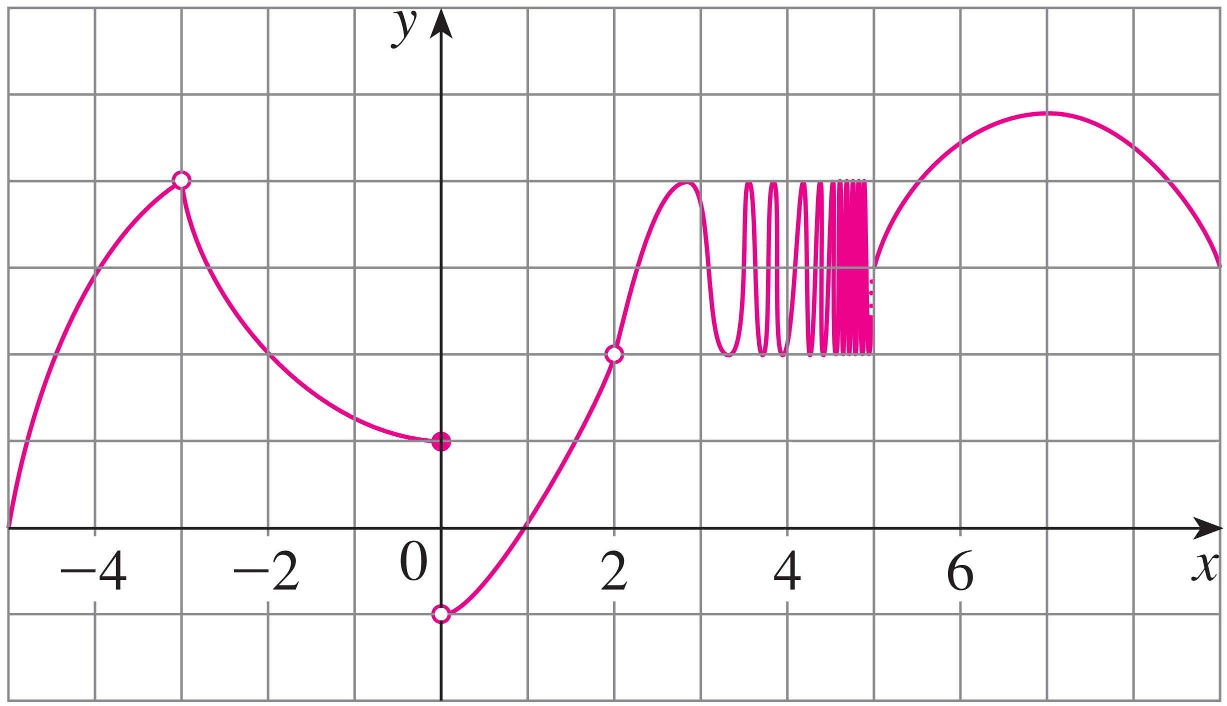
(i) (5 points) Assume that $\tan\theta = 4$, where $\pi \le \theta < 3\pi/2$. Find $\sin\theta$, $\cos\theta$, $\sec\theta$, $\csc\theta$, $\cot\theta$, and $\sin 2\theta$.

Hint: $\sin 2\theta = 2\sin\theta\cos\theta$

(ii) (5 points) You are given a point on the unit circle centered at the origin. Find the x coordinate and the values of the six trigonometric functions.

$$P\left(x, -\frac{\sqrt{15}}{4}\right), \quad x > 0$$

4) (10 points) For the function h whose graph is given, state the value of each quantity, if it exists. If it does not exist, explain why.



1. $\lim_{x \to -3^-} h(x)$

2. $\lim_{x \to -3^+} h(x)$

3. $\lim_{x \to -3} h(x)$

4. $h(-3)$

5. $\lim_{x \to 0^-} h(x)$

6. $\lim_{x \to 0^+} h(x)$

7. $\lim_{x \to 0} h(x)$

8. $h(0)$

9. $\lim_{x \to 2} h(x)$

10. $h(2)$

11. $\lim_{x \to 5^+} h(x)$

12. $\lim_{x \to 5^-} h(x)$

5)

(a) (3 points) Use the squeeze Theorem to evaluate the limit.

$$\lim_{x \to 0} \tan x \cos\left(\sin \frac{1}{x}\right)$$

(b) (3 points) Find the limit or show that it does not exist.

$$\lim_{x \to 4} \frac{x - 4}{\sqrt{x} - \sqrt{8 - x}}$$

(c) (4 points) Evaluate the limit. $\displaystyle\lim_{x \to 0} \frac{\sin 3x \sin 2x}{x \sin 5x}$

6)

(a) (1 point each) State whether the following are true or false.

(i) If $f(x)$ and $g(x)$ are continuous at a, then $f(x)+g(x)$ is continuous at a.

(ii) If $f(x)$ and $g(x)$ are continuous at a, then $f(x)/g(x)$ is continuous at a.

(iii) If $f(x)$ is continuous at a, then $f(x)$ is differentiable at a.

(iv) If $f(x)$ is differentiable at a, then $f(x)$ is continuous at a.

(b) (6 points) Find the values of a and b that make f continuous everywhere.

$$f(x) = \begin{cases} \dfrac{x^2-4}{x-2} & \text{if } x < 2 \\ ax^2 - bx + 3 & \text{if } 2 \le x < 3 \\ 2x - a + b & \text{if } x \ge 3 \end{cases}$$

7)

(a) (5 points) Use the limit definition to compute $f'(1)$.

$$f(x) = 2x^2 + 10x$$

(b) (5 points) Find an equation of the tangent line to the curve $y = 2x^2 + 10x$ at the point $(1, 12)$.

8) (2 points each) Compute the derivatives.

(a) $f(x) = \ln(\sin x)$

(b) $g(x) = \ln(xe^{-2x})$

(c) $F(t) = (t^2 - 2t + 1)^3$

(d) $y = e^{x \sec x}$

(e) $y = \dfrac{x^2}{x^2 + 1}$

9)

(a) (4 points) Show that the equation $x^4 - 3x + 1 = 0$ has a real solution in the interval $[1, 2]$. Explain your reasoning completely; however, you do not have to find the exact value.

(b) (2 points) State Rolle's Theorem.

(c) (4 points) Verify that the function satisfies the hypotheses of the Mean Value Theorem on the given interval. Then find all numbers c that satisfy the conclusion of the Mean Value Theorem.

$$f(x) = (x-1)(x-3), \quad [1, 3]$$

10) (10 points) Find the absolute maximum and absolute minimum values of f on the given interval.

$$f(x) = x^3 - 6x^2 + 8, \;\; [1, 6]$$

11) Let $f(x) = x^3 - 27x - 20$.

(a) (5 points) Find the critical points of f.

(b) (5 points) Find the local maximum and minimum values of f.

12) (10 points) Let $f(x) = 2x^4 - 3x^2 + 2$. Determine the intervals on which the function is concave up or down and find the points of inflection.

Workspace