STAT 232 Categorical Data Analysis **Quiz 4** **March 14, 2025**

Your Name:

Duration of the Quiz is 25 minutes. There are two problems, worth 20 points. Show all your work for full credit. Books, notes etc. are prohibited.

(1) Fit a straight line to the five data points in the accompanying table. Give the estimates of β_0 and β_1. Plot the points and sketch the fitted line as a check on the calculations. Also, calculate S^2.

y	3.0	2.0	1.0	1.0	0.5
x	-2.0	-1.0	0.0	1.0	2.0

(2) Suppose that X is a geometric random variable, where $p_X(k|\theta) = (1-\theta)^{k-1}\theta,\ k = 1, 2, \ldots$. Assume that the prior distribution for θ is the Beta pdf with parameters α and β. Find the posterior distribution and Bayes estimator for θ.

Hint: The Beta pdf with parameters α and β is given by

$$f_Y(y) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\, y^{\alpha-1}(1-y)^{\beta-1}, \quad 0 \le y \le 1.$$