Due by 4pm on Friday, March 14. Please leave your homework on the table before class begins on Friday or leave it in the dropbox outside my office. Do not forget to attach the honor code. There are five problems, 20 points each.

(1) Suppose that we conduct independent Bernoulli trials and record Y, the number of the trial on which the first success occurs. The random variable Y has a geometric distribution with success probability p. A beta distribution is again a conjugate prior for p.

(a) If we choose a beta prior with parameters α and β, show that the posterior distribution of $p \,|\, y$ is beta with parameters $\alpha^* = \alpha + 1$ and $\beta^* = \beta + y - 1$.

(b) Find the Bayes estimators for p and $p(1-p)$.

(2) Information about eight four-cylinder automobiles judged to be among the most fuel efficient in 2006 is given in the following table. Engine sizes are in total cylinder volume, measured in liters (L).

Car	Cylinder Volume (x)	Horsepower (y)
Honda Civic	1.8	51
Toyota Prius	1.5	51
VW Golf	2.0	115
VW Beetle	2.5	150
Toyota Corolla	1.8	126
VW Jetta	2.5	150
Mini Cooper	1.6	118
Toyota Yaris	1.5	106

(a) Plot the data points on graph paper.
(b) Find the least-squares line for the data.
(c) Graph the least-squares line to see how well it fits the data.
(d) Use the least-squares line to estimate the mean horsepower rating for a fuel-efficient automobile with cylinder volume 1.9 L.

(3) A study was conducted to determine the effects of sleep deprivation on subjects' ability to solve simple problems. The amount of sleep deprivation varied over 8, 12, 16, 20, and 24 hours without sleep. A total of ten subjects participated in the study, two at each sleep-deprivation level. After his or her specified sleep-deprivation period, each subject was administered a set of simple addition problems, and the number of errors was recorded. The results shown in the following table were obtained.

Number of Errors (y)	8,6	6,10	8,14	14,12	16,12
Number of Hours without Sleep (x)	8	12	16	20	24

(a) Find the least-squares line appropriate to these data.
(b) Plot the points and graph the least-squares line as a check on your calculations.
(c) Calculate S^2.

(4) (a) Derive the following identity:

$$\text{SSE} = S_{yy} - \hat{\beta}_1 S_{xy}.$$

(b) Use the computational formula for SSE derived in part (a) to prove that $\text{SSE} \leq S_{yy}$.

(5) (a) Suppose that $Y_1, Y_2, \ldots, Y_n$ are independent normal random variables with $E(Y_i) = \beta_0 + \beta_1 x_i$ and $\text{Var}(Y_i) = \sigma^2$, for $i = 1, 2, \ldots, n$. Show that the maximum-likelihood estimators (MLEs) of β_0 and β_1 are the same as the least-squares estimators in the handout on linear models.

(b) Find $\text{Cov}(\hat{\beta}_0, \hat{\beta}_1)$. Use this answer to show that $\hat{\beta}_0$ and $\hat{\beta}_1$ are independent if $\sum_{i=1}^{n} x_i = 0$.

(c) Find the MLE of σ^2.