Due by 4pm on Friday, February 7. Please leave your homework on the table before class begins on Friday or leave it in the dropbox outside my office. Do not forget to attach the honor code. Each problem is worth 20 points.

(1) Suppose that $E(\hat{\theta}_1) = E(\hat{\theta}_2) = \theta$, $\text{Var}(\hat{\theta}_1) = \sigma_1^2$, and $\text{Var}(\hat{\theta}_2) = \sigma_2^2$. Consider the estimator $\hat{\theta}_3 = a\hat{\theta}_1 + (1-a)\hat{\theta}_2$.

  (a) Show that $\hat{\theta}_3$ is an unbiased estimator for θ.

  (b) If $\hat{\theta}_1$ and $\hat{\theta}_2$ are independent, how should the constant a be chosen in order to minimize the variance of $\hat{\theta}_3$?

(2) Suppose that Y_1, Y_2, Y_3 denote a random sample from an exponential distribution with density function

$$f(y) = \begin{cases} \frac{1}{\theta} e^{-y/\theta} & y > 0, \\ 0 & \text{elsewhere} \end{cases}$$

Consider the following five estimators of θ:

$$\hat{\theta}_1 = Y_1, \quad \hat{\theta}_2 = \frac{Y_1 + Y_2}{2}, \quad \hat{\theta}_3 = \frac{Y_1 + 2Y_2}{3}, \quad \hat{\theta}_4 = \min(Y_1, Y_2, Y_3), \quad \hat{\theta}_5 = \overline{Y}.$$

  (a) Which of these estimators are unbiased?

  (b) Among the unbiased estimators, which has the smallest variance?

(3) Let $Y_1, Y_2, Y_3, \ldots, Y_n$ be a random sample with $E(Y_i) = \mu$ and $\text{Var}(Y_i) = \sigma^2$. Show that

$$S^2 = \frac{1}{n} \sum_{i=1}^{n} (Y_i - \overline{Y})^2$$

is a biased estimator for σ^2 and that

$$S^2 = \frac{1}{n-1} \sum_{i=1}^{n} (Y_i - \overline{Y})^2$$

is an unbiased estimator for σ^2.

(4) (a) Is America's romance with movies on the wane? In a Gallup Poll of $n = 800$ randomly chosen adults, 45% indicated that movies were getting better whereas 43% indicated that movies were getting worse.

    (i) Find a 98% confidence interval for p, the overall proportion of adults who say that movies are getting better.

    (ii) Does the interval include the value $p = 0.5$? Do you think that a majority of adults say that movies are getting better?

  (b) For a comparison of the rates of defectives produced by two assembly lines, independent random samples of 100 items were selected from each line. Line A yielded 18 defectives in the sample, and line B yielded 12 defectives.

    (i) Find a 98% confidence interval for the true difference in proportions of defectives for the two lines.

    (ii) Is there evidence here to suggest that one line produces a higher proportion of defectives than the other?

(5) The administrators for a hospital wished to estimate the average number of days required for inpatient treatment of patients between the ages of 25 and 34. A random sample of 500 hospital patients between these ages produced a mean and standard deviation equal to 5.4 and 3.1 days, respectively. Construct a 95% confidence interval for the mean length of stay for the population of patients from which the sample was drawn.