Idempotents in Quandle Rings

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A *quandle* is a set Q with a binary operation $* : Q \times Q \to Q$ satisfying:

(i) For all $x \in Q$, $x * x = x$
(ii) For all $y \in Q$, the map $\beta_y : Q \to Q$ defined by $\beta_y(x) = x * y$ is invertible.
(iii) For all $x, y, z \in Q$, $(x * y) * z = (x * z) * (y * z)$.

The three axioms of a quandle algebraically encode the three Reidemeister moves in knot theory. Let R be an associative ring with unity, and $R[Q]$ be the set of all formal finite R-linear combinations of elements of Q:

$$R[Q] := \Big\{ \sum_i \alpha_i x_i \,|\, \alpha_i \in R, \, x_i \in Q \Big\}$$

Then, $R[Q]$ is a non-associative ring with coefficients in R. We are primarily interested in classifying idempotents in quandle rings $\mathbb{F}_p[Q]$. The Gröbner basis technique plays a pivotal role in our classification. Moreover, we will present a conjecture regarding the idempotents in quandle rings.

This is a joint work with Neranga Fernando.